VAN NOSTRAND'S

ENGINEERING MAGAZINE.

NO. CLXVI.—OCTOBER, 1882.—VOL. XXVII.

HOUSE DRAINAGE AND SANITARY PLUMBING.

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Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

I.

MANY erroneous ideas still prevail about sewer gas and its danger to health which arises, by having so-called "modern conveniences" in our dwellings. It is the purpose of this paper, without in any way adding to the "plumbing scare," clearly to define wherein the danger consists, but at the same time to establish rules for the proper draining and plumbing of houses, which, if carefully observed, will secure to the anxious house owner work of superior quality and of a positively safe character.

Plumbing fixtures, which were considered a luxury years ago, are now believed to be necessary, not only for comfort and convenience, but also, and even more so, for health and for cleanliness. Even a small house is nowadays generally provided with a kitchen sink, a water closet, and sometimes a bath tub, while in a costly modern residence, arranged with an elaborate system of plumbing, we find kitchen, pantry and scullery sinks, slop sinks, laundry tubs, stationary wash basins in closets near bedrooms, a great number of bath or dressing rooms, with water closets, urinals, bath and foot tubs, bidets and other fixtures.

The suggestions and recommendations of this report apply with equal force to the drainage and plumbing of tenements, small houses, costly residences, villas, apartment houses, hotels, factories, school-houses or public buildings. As every plumbing fixture is not only an outlet for the waste water to the drain, but possibly may become an inlet for drain air, the danger increases with the number of fixtures. A multitude of fixtures requires a large number of soil and waste pipe stacks, and the chance of leakage of sewer gas through defective joints increases correspondingly. But be the house large or small, its drainage and plumbing system should always be so arranged as entirely to exclude any possibility of the escape of sewer gas.

SEWER GAS.

I shall, first, briefly consider what is meant by the term "sewer gas." This term, as Prof. W. Ripley Nichols has truly said,* is "an unfortunate one, and gives rise to a quite widespread but very erroneous idea. Many seem to suppose the 'sewer gas' to be a distinct gaseous substance, which is possessed of marked distinguishing characteristics, which fills the ordinary sewers and connecting

* See Prof. W. Ripley Nichols' report upon chemical examination of the air of the Berkley street sewer, in Boston, Mass., 1878.

drains, and which, as a tangible something, finds its way through any opening made by chance or by intention, and then, and only then, mixes with the atmospheric air."

Sewer gas is a mechanical mixture of a number of well known gases, having their origin in the decomposition of animal or vegetable matter, with atmospheric air. This mixture is continually varying, according to the more or less advanced stage of putrefaction of the foul matters, which form a sediment and a slimy coating of the inner surfaces in drains and pipes. It is also variable with the character of this sediment or deposit, and with the physical conditions (moisture, heat, etc.) under which the decomposition takes place.

The principal gases found in sewers and drains are oxygen, nitrogen, carbonic dioxide, carbonic oxide, ammonia, carbonate of ammonia, sulphide of ammonium, sulphuretted hydrogen and marsh gas.

The three first-named gases are the principal constituents of the atmosphere, surrounding the globe, and are found present in the following *average* proportion, viz.:

20.9 vols. oxygen
79.1 vols. nitrogen } in 100 vols. of air, together with 2 to 5 vols. carbonic dioxide in 10,000 vols. of air.

According to R. Angus Smith the amount of *oxygen* is:

In the average, 20.96 vols. in 100 vols. of air.
In pure mountain air, 20.98 vols. in 100 vols. of air.
At the sea shore, 20.999 vols. in 100 vols. of air.
In streets of populous cities, 20.87 to 20.90 vols in 100 vols. of air.

The air in sewers and drains contains much less oxygen, as some of it combines with the carbon of putrefying organic matter forming carbonic dioxide. The amount of nitrogen in the air of sewers is little different from that in the atmosphere which we breathe; but the amount of carbonic dioxide present is greatly increased.

The lowest amount of oxygen in sewer air is recorded to be 17.4 vols. in 100 vols. of air; the amount of carbonic dioxide is in the *average* 2.3 vols. in 100 vols. Sulphuretted hydrogen varies greatly, but the quantity is generally so small as not to be easily determined. Still more difficult is it to find by chemical analysis the proportion of other gases of decay.

In well ventilated and well flushed sewers, Dr. Russell, of Glasgow, found the following ratio:

20.70 vols. of oxygen in 100 vols. of air.
78.79 vols. of nitrogen in 100 vols. of air.
0.51 vols. of carbonic dioxide in 100 vols. of air.
No sulphuretted hydrogen in 100 vols. of air.
Traces of ammonia in 100 vols. of air.

Carbonic oxide is present only in excessively minute quantities, and even then it may have entered the sewer or drain through leakage of illuminating gas from gas mains.

In the absence of more satisfactory methods of analysis, it is usual with chemists to determine the amount of pollution of the air, or the organic matter in it, by determining the amount of carbonic dioxide present, assuming that there is a certain fixed proportion between the amount of carbonic dioxide and the organic matter.* Thus, Prof. W. Ripley Nichols records as the average of many carefully conducted experiments in Boston, the amount of carbonic dioxide in a sewer in that city as follows:

The average of

31 determinations in January, 1878, was 8.7 vols. of CO_2 in 10,000 vols. of air.
44 determinations in February, 1878, was 8.2 vols. of CO_2 in 10,000 vols. of air.
47 determinations in March, 1878, was 11.5 vols. of CO_2 in 10,000 vols. of air.
12 determinations in April, 1878, was 10.7 vols. of CO_2 in 10,000 vols. of air.
8 determinations in June, 1878, was 27.5 vols. of CO_2 in 10,000 vols of air.
8 determinations in July, 1878, was 21.9 vols. of CO_2 in 10,000 vols. of air.
6 determinations in August, 1878, was 23.9 vols. of CO_2 in 10,000 vols. of air.
7 determinations in January, 1879, was 8.0 vols. of CO_2 in 10,000 vols. of air.
14 determinations in February, 1879, was 11.6 vols. of CO_2 in 10,000 vols. of air.
20 determinations in March, 1879, was 11.8 vols. of CO_2 in 10,000 vols. of air.

He remarks: "It appears from these examinations that in such a sewer as the

* Such is strictly true only for air fouled by respiration, while it may not give accurate results in other cases.

In regard to this interesting question I must refer to the Report of Prof. Ira Remsen on the subject of organic matter in the air, published in the National Board of Health Bulletin, vol. 2, No. 11.

one in Berkeley street, which, being of necessity tide-locked, is an example of the worst type of construction, the air does not differ from the normal standard as much as many, no doubt, suppose. In a general way, as we have seen, there is a larger amount of variation from normal air during the warmer season of the year; but even when the amount of carbonic acid was largest, it was only extremely seldom that sulphuretted hydrogen could be detected." . . . . "I think it should be said that the soil pipes and house drains are much more likely causes of discomfort and danger than the sewers."

Hence the importance of a thorough ventilation of all the soil, waste and drain pipes in a building.

Are the above-named constituents of sewer air the origin or cause of the sickness so commonly attributed to the inhaling of sewer gas?

Although many of the gases named are poisonous, if inhaled into the system in large quantities, and may, even if present in smaller quantity, cause nausea, asphyxia, headache, vomiting. etc., none of them can be said to *produce* any of the so-called "filth-diseases." To determine the exact origin of these is a still unsolved problem of physiology. While some believe that the particles of decomposing organic matter, present in sewer air and known as "organic vapor" cause disease, others seek the origin of the latter in microscopic *spores* or *germs* which live and feed upon such organic vapor and are capable of reproduction under favorable conditions, such as presence of putrefying filth, excess of moisture, heat, lack of oxygen, etc.

Whatever theory may be accepted as true, it is evident that, by preventing the decay of organic matter within sewers, drains and soil pipes, or by depriving these germs (if such be the cause of disease) of the conditions facilitating their reproduction, we can best prevent the outbreak of excremental diseases. In other words, *by completely removing as speedily as possible all waste matters from the dwelling by pipes thoroughly and tightly jointed, and by a sufficient dilution of the air in these pipes with oxygen, the danger of infection, arising from defective drainage and plumbing, may be reduced to a minimum.*

It should be mentioned that some hygienists, notably Dr. Soyka and Dr. Renk, both assistants of Pettenkofer in Munich, have lately denied the existence of any positive proof of a connection between sewer gas and the spread of epidemic diseases—just as Naegeli and Emmerich doubt the possibility of infection from drinking water contaminated by sewage. Dr. Renk considers the exclusion of gases of decay from the interior of dwellings necessary only so far as they are offensive to the sense of smell. In this view, however, I cannot concur; in regard to "filth-diseases," their causes and origin, I accept the theory of Dr. Simon, Parkes and others.

DEFECTIVE AND GOOD PLUMBING WORK.

The unhealthiness of dwelling houses has been greatly increased by plumbing work defective in design, materials and in workmanship, through ignorance, but often through intention of builders. The consequence was a growing inclination with some to abandon all plumbing fixtures, to go back to the ill-famed privy in the backyard, and to follow the practice of throwing the slops from the kitchen upon the grounds in the rear yard.

But, cannot this risk be avoided with careful, conscientious and honest workmanship, carried out under the strict supervision of an expert? Is it such a difficult thing to have a proper and judicious arrangement of the drainage system?

I shall endeavor in the following pages to explain what the elements of a well devised system of house drainage and sanitary plumbing are. Much has been written of late about this subject. It has been well and thoroughly treated by able writers, and my paper can hardly claim much originality or novelty, but should be taken as the outgrowth of much study and experience.

The essentials of a perfect system of house drainage are simple and can be readily understood by any householder, when carefully explained. They involve nothing more than the proper application of well-known laws of nature; there is no mystery, no secrecy about any part of the work. Any one building a house is able to secure good drainage and a safe arrangement of the plumbing work without having to resort to any patented system. The proper way of laying and

trapping drains, of ventilating soil and waste pipes, etc., cannot, in my judgment, be patented. The plumbing fixtures are, of course, mostly patented, as any useful appliance may be, and in speaking of these one cannot avoid recommending patented devices.

The entire sewage of the dwelling may deliver either into a regular system of sewers, or else discharge into an open water course; or—in the absence of either—it may run into a cesspool, be it a leaching cesspool, or a well-cemented, tight vault of brickwork; or finally, into a flushtank, to be disposed of on the ground by surface irrigation, or below the ground by the subsurface irrigation system.

So far as the arrangement of the *inside* plumbing work is concerned, it does not make any material difference which of the above systems of getting rid of the waste-water from habitations is available.*

Under all circumstances the three cardinal objects to be fulfilled by a perfect system of house drainage are:

1. To remove from the inside of the dwelling as quickly as possible all liquid and semi-liquid wastes, whether it be the soapy discharge from wash bowls, bath tubs and laundry tubs, or the vegetable refuse from the scullery sink, the greasy matter from kitchen and pantry sinks, or the foul discharges from slop sinks, urinals and water closets.

2. To prevent the foul gases originating from the decomposition of the above matters in the drain, sewer, cesspool or flushtank, from returning through the same channels into our dwellings.

3. To oxidize and render inocuous by a copious flushing with air the foul gases due to the possible putrefaction of waste matters within the house drains, soil and waste pipes, at the same time properly protecting all outlets of fixtures from the entrance of these gases.

DRAINS OUTSIDE OF THE HOUSE.

The house drain is the means for conveying the sewage from the dwelling. Its proper material is a question of great importance. Outside of the dwelling it should be of vitrified pipe, circular in shape, which is superior to cement pipe. Iron pipe for outside drains is preferable in made ground, or in quicksand, also where trees are near the line of the drain, and where the drain must necessarily pass near a well furnishing water for the household. Neither brick channels nor wooden conduits should be used for this purpose. Only strong, hard, well-burnt, vitrified pipe, free from cracks or other defects should be used. Four inch pipes and those of smaller size are especially liable to warping, and should be carefully inspected and selected. The interior of these pipes should be well-glazed and smooth throughout; the pipes should be impervious, true in section, perfectly straight, and of a uniform thickness. Four inch pipes should have a thickness of $\frac{1}{2}$ in. to $\frac{5}{8}$ in.; six inch pipes $\frac{11}{16}$ in. to $\frac{3}{4}$ in.; nine inch pipes should be not less than $\frac{3}{4}$ inches thick; 12 inch pipes should be 1 inch thick; fifteen inch pipe $1\frac{1}{4}$ in., and eighteen inch pipe should have a thickness of $1\frac{1}{2}$ inches.

The joints of the pipes should receive particular attention. The danger arising from imperfect or leaky joints is twofold, namely, first, the sewage, by soaking into the ground, pollutes the soil and endangers the purity of the water supply in places where houses are dependent on wells end cisterns for water. The ground around and under the house is more and more subject to contamination, and in winter time, when there is a strong inward draft into houses from fireplaces and stoves, the tainted "ground air" is thus sucked into our very living and sleeping rooms, often producing severe illness. The second danger resulting from leaky joints is equally patent. The solid matters, carried in suspension in the pipes, are deprived of a part of their liquid carrier, and thus tend to accumulate and form deposits in the house drain, which deposits soon undergo decomposition, and fill the drains and pipes with noxious gases.

Vitrified pipes are made either with a socket or hub attached to one end of the pipe, or with both ends plain. When socket pipe is used, special grooves should be cut in the bottom of the trench for the hub, in order to give the pipe a solid bearing on its entire length. The pipes are laid with the socket pointing upgrade, the plain or spigot end of one

* It is not intended in this paper to discuss the merits and faults of these different methods of sewage disposal.

pipe being inserted into the socket of the next. Spigot and socket ends should be concentric. Into the annular space between both a gasket of picked oakum is introduced and firmly rammed by a hand iron. The remainder of the space is then filled with pure cement, or cement mixed with an equal volume of sand. No lime should be used with the mortar, which should be prepared only in small quantities at a time, to prevent its setting before use. Particular attention should be given to the bottom part of the joint, where the mortar should be pressed into it with the fingers. If water accumulates in the trench, this should be carefully removed from the grooves before making the joints, and sufficient earth should be thrown into the groove to support the mortar at the bottom of the joint, until it has time to harden. The gasket of oakum prevents any cement from projecting into the inside of the drain, and renders the use of a rattan and rag, with which to wipe the inside of joints, unnecessary. Where the sockets are insufficient in length to permit the use of a gasket, it becomes important to clean the joints of cement projecting at the inside, but in this case a better device than a rattan with rag tied to it is a strong handle to which is attached a a semi-circular disc of wood, of a somewhat smaller radius than the radius of the pipe.

The cylindrical pipe without sockets is preferred by some. The joints, in this case, are made by butting two pipes together, and covering them with rings or collars of unglazed terra cotta, applying cement to the inside of the collar and to the ends of the pipes.

Some object to the use of cement for drain pipe joints, claiming that the stiffness of the cement joint after hardening will tend to break the pipes in case of a slight settling. They also maintain that some cements increase considerably in volume when setting, and tend to burst the sockets. They much prefer a ring of puddled clay, pressed into the joint and wiped around it, claiming that clay will make a tight and more elastic joint. But in ordinary cases the settling of drain pipes may be prevented by providing a solid foundation of either gravel, sand, or concrete, or in very wet ground, boards or piles as supports to the pipe. In made ground I should recommend the use of iron pipes to prevent leaky joints or breakage of pipes. A good Portland cement will not much increase in volume after setting, and I believe it has been shown that those cements which largely increase their volume, often lose their hardness after some time, and would be, therefore, unfit for any use. While I fully appreciate the advantage of a somewhat elastic joint, I do not think that puddled clay will make as tight a joint as seems desirable for drains carrying foul sewage.

What is known as "Stanford's Improved Pipe Joint" has been used extensively of late in works of house drainage in England, and its superior merits are such as to recommend it for use with us. I, therefore, introduce a brief description. "In sewer work in bad or wet ground, just where a sound joint is most required, the difficulty of making it is the greatest. What is wanted, therefore, is a joint that will entail the least disturbance of the ground, that will not necessitate the absolute drying of the trench bottom, and that will require the minimum of time, skill, and labor in making it. These conditions will be fulfilled in the most complete manner by making the spigot of one pipe to fit mechanically into the socket of another, as in a bored and turned iron pipe joint. Such a mechanical fit cannot be obtained with stoneware or earthenware pipes, owing to the difficulty of preserving perfect accuracy of form during the process of burning."

"In the Stanford joint tightness is obtained by casting upon the spigot and in the socket of each pipe, by means of moulds prepared for the purpose, rings of a cheap and durable material, which, when put together, fit mechanically into each other, and by making these rings of a spherical form, a certain amount of movement or settlement may take place without destroying the accuracy of the joint. In laying these pipes, therefore, all that is necessary is to insert the spigot of one fairly and firmly into the socket of another previously laid, and the joint is complete and perfectly watertight. A smearing of some kind of grease is frequently found to be of advantage."

Half-socket or access-pipes are some-

times useful, where it becomes necessary often to inspect the house drain. They should be located close to angles, bends, junction branches, running traps, &c. They are not much used in this country, owing, probably, to the fact that, should the main drain run over one-half full, sewage may leak out through the access-pipes into the soil.

Care should be taken to lay the pipes on a firm bed of sand or gravel, and if this is not available, a concrete base should be provided in the trench. The pipes should be laid in straight lines, all changes of direction should be effected by curves of as large a radius as possible, formed of bent pipes. All branches should join the main under an acute angle, by special Y pieces, for a right-angled junction (by a T branch) tends to form eddies and consequently deposits in the main drain.

In laying drains, care should be taken to avoid, as much as possible, trees. The roots of these are frequently found to penetrate and often choke the pipes, and are certainly a dangerous obstruction to the flow in the drain. If the line of the drain must necessarily pass near trees, the use of iron pipes is recommended. The coating of the pipes with coal tar on their outside, the use of asphaltum for joints, and sometimes the surrounding of the drain with a strong layer of concrete are said to be effectual protections against roots of trees.

I now must speak of the *grade of the drain*, as this is a matter of prime importance. Upon the inclination of a pipe depends the *velocity* of the water flowing through it. If this velocity should be insufficient, deposits will occur, and the drain will in time become choked. Pipes of 4 inches diameter should have a velocity of flow of from 3 to 4½ ft. per second; those of 6 and 9 inches diameter should have a velocity of not less than 2½ to 3 ft. A velocity of 2 ft. per second should be considered the minimum allowable in house drains. As a general rule the inclination of a house drain should be as great as attainable, and must be, wherever local conditions will permit, continuous. It is not unfrequently found by uncovering old drains that, in order to save digging, they are laid very flat, often perfectly level, from the point where they leave the house to nearly their junction with the sewer, at which place they are turned with a steep pitch downwards, and often enter the sewer at its crown. By distributing the whole available fall over the total length of the drain a much better grade would have been secured.

In order to lay a drain with a true grade, especially where the fall is little, a level should be used. The elevation of bottom of pipe, where it leaves the house—at a depth of not less than 3 feet in the New England States, as a protection against frost—should be ascertained, as well as the elevation of the junction with the sewer (or else inlet to cesspool or flush tank). A profile of the ground along the line of the drain should also be determined by levelling. Thus, the proper available fall can be determined, with a little additional trouble, it is true, which, however, will be well repaid by securing a much better quality of the work.

A fall of from 1 in 40 to 1 in 60 is desirable for pipes of 4 or 6 inches diameter, but this cannot always be had. I would consider a grade of 1 in 100 as the least to be given to house drains, in order to keep them self-cleansing. When laid with such fall and running full or half-full, a six-inch drain has a velocity of 3½ feet, a four-inch drain a velocity of nearly 3 feet, which is sufficient to carry along such suspended matters as only ought to enter a house drain. Where the available fall is less than 1 in 100, special flushing apparatus, such as Field's flush tank, McFarland's tilting tank, or Shone's hydraulic syphon ejector should be used.

I have thus fully explained the right method of laying drain pipes, because, even with the best plumbing inside of the house, it is of the greatest importance to have the outside drains of good quality, properly laid, and properly jointed.

The next question to be considered is: *What is the proper size for house drains?*

This will, of course, depend to some extent upon the grade of the drain, the size of the house and number of its occupants, the amount of water used per head per day, and finally, unless the rain falling upon the roof is stored in a cistern, upon the amount of rainfall to be

carried off in a certain time. This rain is a most beneficial scourer for drains, and unless the sewage of the dwelling is to be disposed of by irrigation, or the sewers of the town built according to the "separate system," which excludes the rain-fall from the channels carrying sewage, I should strongly advise to deliver it into the same channel, which carries away the foul wastes of the habitation. Even with this double purpose in view the house drain need not be very large, and the closer its size is proportioned to the volume of water it must carry the more *self-cleansing* will it be.

To illustrate the advantage gained by

Fig. 1.

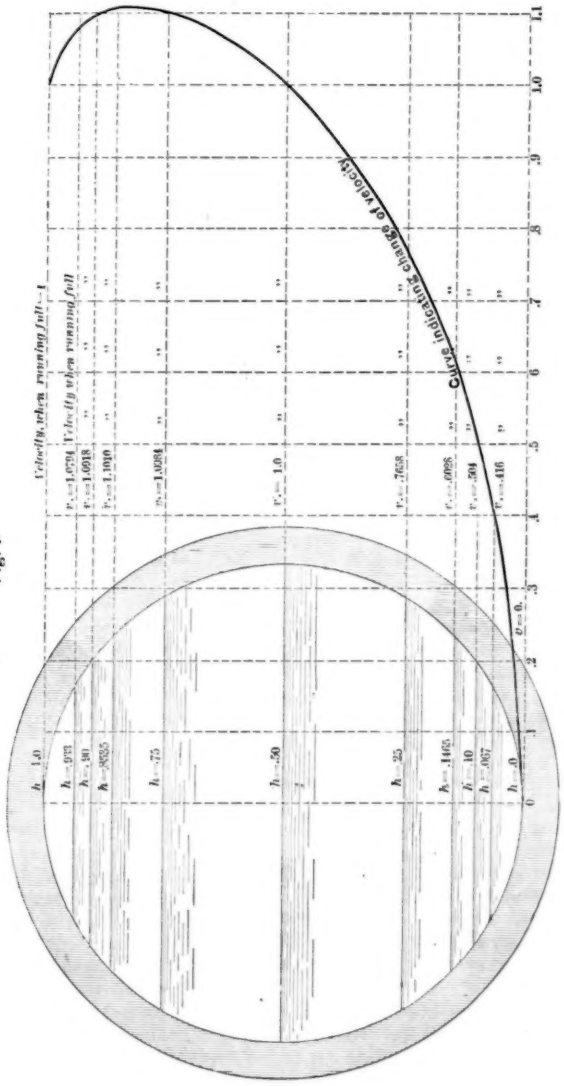


DIAGRAM REPRESENTING CHANGES OF VELOCITY IN A CIRCULAR SEWER OF THE DIAMETER = 1, LAID AT A GIVEN GRADE, AT DIFFERENT DEPTHS OF FLOW.

reducing the size of drains as much as possible, or in other words by concentrating the sewage flowing through it, I have constructed the diagram, Fig. 1, which represents for different depths of flow in the same pipe the change of velocity. It is evident that the velocity in a pipe will greatly diminish as the depth of the stream flowing through it diminishes. The diagram shows that the velocity is the same for drains running full or half full; it also shows that the maximum velocity of flow occurs not when the sewer is running full, but when the depth of flow is about .813 of its diameter. The maximum velocity is about 11 per cent. greater that that of a pipe running full or half full. The maximum discharge, however, does not coincide with the maximum velocity. The discharge is a maximum when the depth of flow is about .95 of the diameter. At a depth of flow of one fourth of the diameter the velocity is only about 77 per cent. of that when running full or half full, and for lesser depths of flow it diminishes rapidly.

For an ordinary city dwelling a drain four inches in diameter is ample, even including all the rain-fall. For a larger lot and residence a six-inch drain is all that is needed, even if the fall should be only 1 in 100. As a general rule, house drains have been constructed of too large a diameter, and one often meets with the objection that a four-inch pipe will clog up with grease in a short time, or will be obstructed by solid substances. To this, I answer, that in regard to grease the only safe way, where it is allowed to waste, or in case of large boarding-houses and hotels, is to keep it altogether out of the drain (which can be easily accomplished by a suitable grease trap). Grease congealing in a drain is sure to clog it, no matter how large it is made. The stoppage would be only a question of time, and nothing could be gained by postponing this inevitable result. In regard to obstructions by solid matters, I may assert that nothing which passes through the strainer of a sink or from the water-closet bowl can possibly obstruct the drain. What may enter through carelessness of servants, or of the householder, such as "sand, shavings, sticks, coal, bones, garbage, bottles, spoons, knives, forks, apples, potatoes, hay, shirts, towels, stockings, floor-cloths, broken crockery, etc.," to quote from Mr. J. Herbert Shedd's Report on the Sewerage of Providence, cannot rightfully be expected to be carried away in a drain. To guard against such obstructions, the drain should be made accessible, especially near bends, junctions and the main trap.

The following useful table, calculated by Robt. Moore, Esq., C.E., from Weisbach's formula for flow of water through open culverts, gives the size and velocity in house drains, laid at different inclinations, and for various sizes of lots, the rain-fall being 2 inches per hour, and the pipes running $\frac{3}{4}$ full. It should be said that the smallest sizes of the table (below 3 or 4 inches diameter) are given only for the sake of completeness, and not as sizes to be recommended for actual use.

Take, for example, an ordinary city lot of 25×150 ft. $= .0861$ acres. The rain-fall to be provided for may be 2 inches per hour. Though such storms are not frequent, provision should be made for them in the calculation of the size of house drains, as the rain falling on roofs and on paved yards reaches the drain very soon after having fallen. A rainfall of 1 inch per hour per acre very nearly yields 1 cubic foot per second, therefore 2 inches per hour give 2 cub. ft. per sec. per acre. The number of cubic feet of rain from the above lot is therefore $.0861 \times 2 = .1722$ cub. ft. per second or $60 \times .1722 = 10.332$ cub. ft. per minute.

We further assume 6 persons to the house, and 75 gallons per head per diem, which is a very liberal allowance. The waste water of the house is therefore $6 \times 75 = 450$ gallons per day. If one-half of this amount is estimated to run off in 8 hours, the maximum per hour would be about 28 gallons or .0624 cub. ft. per minute. This quantity is so insignificant compared with the rainfall that we may safely neglect it.

Should the drain be allowed to run three-quarters full, and have a fall of 1 in 100, a diameter of $3\frac{3}{4}$ inches would suffice, according to above table.

As a second example, I shall take a large lot, say 80×150 ft. $= .2755$ acres. The quantity of rain to be discharged will be, under the same suppositions as above, $2 \times 60 \times .2755$ acres$=33.06$ cub. ft. per minute. For a drain, running $\frac{3}{4}$

Table of Diameters of House Drains

With various Grades, and for Lots of different sizes, capable of discharging 2 inches of rain per hour when running three-fourths full.

Calculated by Robert Moore, C. E., St. Louis, Mo.

Dimensions of lot in feet.	No. of acres.		Fall, 1 per 100.	Fall, 1½ per 100.	Fall, 2 per 100.	Fall, 2½ per 100.	Fall, 3 per 100.	Fall, 4 per 100.	Fall, 5 per 100.
20x150	0.0689	Velocity.....	2.69	3.16	3.54	3.87	4.17	4.68	5.11
		Diam. Inches	3½	3⅛	3	2⅞	2⅝	2⅜	2¼
25x150	0.0861	Velocity.....	2.81	3.30	3.71	4.05	4.36	4.89	5.35
		Diam. Inches	3¾	3½	3¼	3⅛	3	2⅞	2⅝
30x150	0.1033	Velocity.....	2.91	3.43	3.84	4.20	4.52	5.07	5.54
		Diam. Inches	4	3¾	3½	3⅜	3¼	3	3
35x150	0.1205	Velocity.....	3.00	3.53	3.96	4.33	4.66	5.23	5.72
		Diam. Inches	4¼	4	3¾	3⅝	3½	3¼	3⅛
40x150	0.1377	Velocity.....	3.09	3.59	4.07	4.45	4.79	5.37	5.87
		Diam. Inches	4½	4¼	3⅞	3¾	3⅝	3½	3¼
45x150	0.1550	Velocity.....	3.16	3.71	4.17	4.56	4.90	5.45	6.01
		Diam. Inches	4¾	4⅜	4⅛	4	3⅞	3⅝	3½
50x150	0.1722	Velocity.....	3.23	3.79	4.26	4.65	5.01	5.62	6.14
		Diam. Inches	5	4½	4¼	4⅛	4	3¾	3⅝
60x150	0.2066	Velocity.....	3.35	3.93	4.41	4.88	5.19	5.83	6.37
		Diam. Inches	5⅛	4⅞	4⅝	4⅜	4¼	4	3⅞
70x150	0.2410	Velocity. ...	3.45	4.06	4.55	4.98	5.35	6.01	6.57
		Diam. Inches	5⅜	5¼	4⅞	4⅝	4½	4¼	4⅛
80x150	0.2755	Velocity.....	3.54	4.17	4.68	5.11	5.50	6.17	6.75
		Diam. Inches	6	5½	5¼	5	4⅞	4½	4⅜
90x150	0.3099	Velocity.....	3.63	4.27	4.79	5.23	5.63	6 32	6.91
		Diam. Inches	6¼	5¾	5½	5¼	5	4¾	4½
100x150	0.3443	Velocity.....	3.71	4.36	4.89	5.35	5.75	6.45	7.05
		Diam. Inches	6½	6	5⅝	5½	5¼	5	4¾
125x150	0.4304	Velocity....	3.87	4.56	5.11	5.59	6.01	6.75	7.38
		Diam. Inches	7⅛	6½	6¼	6	5⅝	5⅜	5⅛
150x150	0.5165	Velocity.....	4 02	4.73	5 30	5.80	6.24	7.00	7.65
		Diam. Inches	7⅝	7¼	6⅝	6⅜	6⅛	5⅞	5⅝
175x150	0.6026	Velocity.....	4.14	4.87	5.47	5.99	6.45	7 22	7.89
		Diam. Inches	8⅛	7½	7⅛	6⅞	6½	6¼	6
200x150	0.6887	Velocity.....	4.26	5.06	5.62	6.14	6.61	7.41	8.10
		Diam. Inches	8⅜	8	7½	7⅛	6⅞	6½	6¼

full. the table gives the necessary diameter=5¼ inches.

For a convenient graphical exhibit of the relation between inclination, size, velocity and discharge of drains and sewers see the author's "Diagram for Sewer Calculations," 1881, N. Y.

The foregoing explanations have, I believe, sufficiently proved that *no house drain needs to be larger than six inches*

under ordinary circumstances, while in most cases a 4-inch pipe will fully answer the purpose. Any increase of size would tend to be a detriment rather than a benefit.

DRAINS INSIDE OF THE HOUSE.

The earthenware drain should end at about 5 to 10 ft. outside of the foundation walls of the house. From this point towards the inside of the house the drain should be of iron. The joint between iron drain and earthenware pipe should be made with pure hydraulic cement. Where the iron pipe passes through the wall, a relieving arch should be built over it. Settlement of walls often occurs, and is liable to crack the pipe or even break it, unless the above provision is carried out. It is quite evident that, under no circumstances whatever, this part of the house drain should consist of vitrified pipe.

Important as it is to have the drains outside of the house free from sediment or leakage, it is still more so to have all the pipe joints inside of the dwelling perfectly air and water tight, for if any defect should exist here, sewer gas will leak into the cellar and pervade the whole house. For this reason we sometimes find the cardinal rule laid down that no drains should run under a house, but should be taken outside of it as soon as possible. This is not practicable, as a general rule, in the case of narrow city lots. Fortunately, however, we can, with perfect safety, run the drains across the basement or cellar floor of a dwelling, provided we choose the only safe material, i. e. *iron pipes*. A good mechanic is able to make with these a perfectly air and water tight joint.

The best course of the iron drains in the house is along the ceiling of the cellar, or along one of the foundation walls. In other words, wherever practicable, the iron drain ought to be kept *in sight*, in order to enable anybody to detect a leaky joint at occasional inspections. Sometimes fixtures located in the cellar, such as servants' water closets, laundry tubs or sinks, make it necessary to lay the iron drain below the cellar floor. In this case it should be laid with proper fall in a trench, the sides of which are walled with brick work, and the base of which should consist of a layer of from 4 to 6 inches of concrete, thoroughly rammed and properly graded. The trench should be made accessible by closing it with movable covers of iron or wood.

If the drain is carried in sight, I would much prefer supporting it by strong iron hooks from the cellar wall, or by brick piers, where the ground is solid, and not liable to "settle," instead of suspending it by iron hangers from the main joists of the floor above. For, with the latter arrangement, a slight lowering or bending of the beams supporting the iron drain, would tend to loosen the joint between water-closet trap and soil pipe, as the latter is rigidly connected with the drain, thus creating a source of danger from leakage of sewer gas.

As regards the proper inclination of iron drains in the cellar, the rules given for the outside drains should be observed.

The principles stated for the size of the outside drain apply with equal force to the inside drain. If no leaders enter the drain at its upper end or along its course through the house, a 4-inch pipe is ample for any ordinary sized dwelling; a 6-inch drain is very seldom required.

As a good precaution for repairs or cases of obstructions of the drain, I would recommend the practice of many plumbers, which consists in inserting at distances of about 10 or 20 feet along the course of the iron drain Y branches, the ends of the branches being closed by a brass thimble, caulked into the hub of the Y, and closed by a trap screw. By opening these and inserting a proper cleaning tool, occasional obstructions by introduction of foreign matters are easily removed.

The course of the main drain in cellar should be as straight as possible. All changes of direction should be made by iron bends. All junctions with the main drain should be made by Y branches, in order to join the flow of both pipes without causing eddies; no right-angled junction should be made in any horizontal or inclined pipe.

SOIL AND WASTE PIPES.

Into the iron drain the vertical soil and waste pipes enter by means of either

quarter bends or by a Y branch with an eighth bend.*

The best material for soil and waste pipes is cast iron. All cast iron pipes used in house drainage should be thoroughly sound, of a *uniform* thickness throughout, and must allow of ready cutting without splitting. The inisde should be truly cylindrical and of smooth finish. The thickness of ordinary (so-called *light*) soil pipe is about $\frac{1}{8}$ of an inch for 2, 3 and 4-inch pipes, and $\frac{5}{32}$ to $\frac{3}{16}$ of an inch for 5 and 6 inch pipe. For all large public or private buildings I should always insist upon the use of *extra heavy* soil pipe, which is about double as thick as the ordinary pipe. The weights of extra heavy pipe are about as follows:

2 inch pipe,	5½ lbs. per foot.
3 inch pipe,	9½ lbs. per foot.
4 inch pipe,	13 lbs. per foot.
5 inch pipe,	17 lbs. per foot.
6 inch pipe,	20 lbs. per foot.

Great care should be exercised by plumbers, architects, plumbing inspectors and sanitary engineers in regard to the *uniform* thickness of iron soil pipe. The writer has lately seen specimens of extra heavy soil pipe where the pipe was almost as thin as a knife-blade on one side, while it had far more than the required thickness on the other side, the weight being as specified. Measuring the thickness of iron drain pipes by a pair of calipers should be recommended, but I am not aware that it is done at all now.

Iron soil pipe, the inside of which has been made smooth by dipping the pipe into a hot solution of coal-tar pitch, is superior to ordinary iron pipe. This coating, when applied to the outside of the pipe, forms a good preventive against rust or corrosion, and is better than any paint applied to the iron. Where economy is no object, the *enamelled* pipe may be used, which has a very smooth inside surface, thus securing to well-flushed soil pipes the greatest possible cleanliness. Whether iron pipes are coated with coal tar pitch or enamelled, it is necessary, before applying either of these protective coats, carefully to test each pipe for defects, sand holes or cracks, by the hammer test. The coating may effectually cover these defects and render detection difficult.

Iron pipes are manufactured in lengths of 5 feet, with hub and spigot end, or else with double hub.

The iron works manufacture not only straight soil pipe, but a large number of fittings, such as quarter bends, eighth bends, sixth bends, sixteenth bends, T branches, Y branches, double Y branches, half Y branches, offsets, single and double hubs, increasers, reducers, &c., to enable the plumber to make all possible connections and lines with iron pipe.

In England *lead pipe* is preferred for soil pipes. According to one of the best English authorities on plumbing* the advantages claimed for lead pipe are briefly as follows:

1. It is smoother, cleaner, not so corrosive; more durable.
2. It can be bent to suit any position; it is more compact.
3. Its joints are more to be depended upon than iron pipe joints.
4. Urine, being very corrosive, acts more on iron than on lead.
5. Iron pipe rusts on the outside, and painting iron pipes, to prevent it, is expensive, and is generally not done thoroughly at the back of the pipe.
6. Lead branch wastes or traps cannot easily be joined to iron pipe.

* As regards the exact meaning of the terms drain pipe, soil pipe, and waste pipe, I quote the following clear explanation from the "Sanitary Engineer," Vol. 4: "The drainage system of a house, including the pipes or channels of any kind connecting it with the sewer or cesspool, may be divided into two parts—first, that part which is chiefly outside the house walls, and second, that which is generally inside the house. The first is called the *house drain*, or simply *drain*, and conveys the whole body of wastes from the house, including both the discharges from water-closets and urinals, and from baths, basins, sinks, &c., to the sewer or cesspool. The *drain* is practically *horizontal*, and may be considered as terminating either at the house wall, or at the most remote point at which it receives the pipes from any fixtures. The word *drain* is, however, also used in another sense as distinguished from *sewer*. It then means the pipe or channel which conveys only rain or ground water, as distinguished from sewage. An example of this kind of drain is the separate system of pipes, used to convey only rain water in some towns and the tile pipe commonly employed in draining wet lands.

"That part of the house drainage system which is generally inside the house, including the pipes from the various fixtures, is made up of *soil pipes* and *waste pipes*. *Soil pipes* are those pipes which receive *human excreta* from water closets and urinals, and they are still called soil pipes, even if they also receive the waste water from baths, basins, &c. On the other hand, *waste pipes* are those which receive *only* the waste water from these latter, but not the discharge from water closets and urinals. The *waste* pipes of a house may either enter the *house drain* independently, or join the *soil pipe* first and discharge their contents through it into the *drain*. As distinguished from the *drain* the *soil pipes* and *waste pipes*, at least for the longer lengths, are generally vertical."

* S. Stephens Hellyer, "The Plumber and Sanitary houses," 2d edition.

7. Iron pipe does not allow caulking joints with lead, therefore cement is used for the joint.

From all this I disagree, for:

1. Tarred or enamelled iron pipe is fully as smooth as lead pipe, and the iron pipe is thereby well protected from corrosion.

2. The above enumerated variety of special fittings enables the plumber readily to adapt his iron pipe to almost any position; moreover I do not see why iron pipe should take up a great deal more room than lead pipe of same bore.

3. *Well caulked* joints of heavy iron pipes are just as sound and trustworthy as wiped joints in lead pipes, and any good mechanic is able to make them.

4. Urine does not corrode an iron soil pipe, protected by a coal-tar pitch solution or by enamel, more than a lead pipe.

5. The outside of iron pipe can be efficiently protected from rusting by paint, coal-tar pitch or enamel.

6. Lead cannot be caulked into iron, but a good plumber always solders a brass ferrule by a wiped joint to the lead pipe (or trap), and caulks the brass ferrule into the hub of the iron pipe.

7. Any one who will take the trouble carefully to examine the joints of iron pipe, made by an honest and conscientious plumber, will readily admit the possibility of making tight joints with iron pipe. Only iron pipe of a sufficient strength to withstand the knocking occasioned by caulking the lead is used in American plumbing.

But, while iron pipe is fully equal in all the above respects to lead, it has great advantages over it. "Lead soil pipes are very heavy, and, therefore, liable to sag and split open, to have holes eaten into them by rats, and have nails driven into them by carpenters, and also to corrode, and they require much greater skill to put up, and involve more expense; therefore the statements of Hellyer prove nothing, although they demonstrate the absurdity of bricking soil pipes into a wall, and the necessity of so placing them that they are at all times readily accessible for inspection; and also prove what few people seem to realize, that the drainage system of a house requires periodical testing and inspection just as much as a steam boiler or piece of machinery."*

Pipes of wrought-iron, coated with coal-tar pitch, have been lately used for soil pipes, notably in the Durham system of house drainage. I am not prepared to say whether or not such pipes last as long as cast-iron pipes protected with the same coating.

Soil pipes should not, as a rule, be larger than four inches inside diameter; this size will answer for half a dozen or more water closets on one vertical stack of pipe. From a late account of the sewerage of the city of Pullman, near Chicago, I learn that several hundred soil pipes of 3-inch bore were used in the houses, and "in the case of three-story flats, one pipe frequently has six closets connected to it." Very few instances of stoppage occurred, and these were always "due to obstructions that got in during construction, and never to the use of a small-sized pipe." Such a reduction of the size of soil pipes will undoubtedly increase the danger of "siphonage of traps," and for this reason it is hardly safe to use soil pipes smaller than four inches inside diameter.

Waste pipes of iron should be $1\frac{1}{2}$ or 2 inches in diameter. This is ample for the waste water of one or more bath tubs, and a large number of wash bowls.

I may here remark that, contrary to the generally entertained opinion, a nearly horizontal or inclined pipe can be kept clean by flushing much easier than a vertical pipe. The flushing water in this latter case soon assumes a whirling motion, and the scattered drops fall downward without exerting much scouring action upon the interior of the pipe. Hence the importance of having the inside of soil and waste pipes as smooth as possible to prevent solid matters from adhering to the sides, where hardly any amount of flushing will take them off.

The arrangement of soil and waste pipes should be as direct as possible. It is desirable that each vertical stack should extend from cellar to roof in a straight line. In planning the plumbing for a dwelling too much care cannot be taken to secure such an arrangement.

* See articles on "Plumbing Practice," in the Sanitary Engineer, vol. 4.

Every offset, every bend in the pipe forms an obstruction to its proper flushing, with both water and air. Horizontal soil pipes are especially objectionable; the water closets, baths, bowls and sinks should always be located in groups, and as near to their respective pipes as possible.

It is desirable to run soil pipes and waste pipes in sight, so that they may be accessible. I decidedly condemn the usual plan of architects of building recesses or niches in the walls for pipes. The difficulty of caulking the back part of pipe joints in this position is very great. Where objection exists to having the pipes in sight, they should be boxed up, but I would always insist upon having the cover fastened by screws, which can be easily removed, and not by nails.

Iron soil and waste pipes should be supported at distances of not over five feet by strong iron hangers or hooks.

Branch pipes should enter the vertical stack by means of a Y or half Y branch, wherever possible; a right-angled junction, by a T branch, is not so objectionable here as in the case of horizontal or inclined pipes.

In badly drained houses, with cheap plumbing work, it is not uncommon to find the joints of pipes made only with sand and paper, or with putty, mortar, cement, sulphur and pitch and red lead, or other material. All of these joints are worthless, and therefore extremely objectionable.

Joints of iron pipe should be made by first inserting a little picked oakum into the socket, care being taken that no part of this gasket enters the pipe. The oakum prevents the molten lead from running into the pipe, where it might form an obstruction to the flow. Molten lead is then poured into the hub, enough quite to fill it. As lead shrinks in cooling, it must afterwards be carefully hammered with a special caulking tool, thus filling the space between spigot and hub, so as to make a perfectly gas and water tight joint. In order to be able, at all times, to inspect the joints, it is a good practice to leave the caulked lead without a cover of paint, cement or putty, the marks of the caulking tool being thus left exposed to view.

A tight joint can also be made with a mixture of sal ammoniac, iron filings and sulphur. Such "rust joints," however, are not much used for soil pipes.

Where wrought-iron is used for soil and waste pipes, the joints are screw joints, and can be made tight as in steam fitting work.

When all the iron piping in the house is completed, the tightness of the joints should be thoroughly tested, before connecting the fixtures. The test which is mostly used, is the "water pressure test." The end of the iron pipe outside of the foundation walls is tightly closed by a wooden plug, or better, a disc of india rubber, which can be squeezed between two iron discs. All branches of soil pipes and waste pipes are similarly closed. The pipes are then filled with water, which must stand in them for some time. If the subsequent inspection shows a lowering of the water level, there must be a leak at some joint, or else some defect exists in the iron piping. Of course the leak must be found and repaired, and the test should then be repeated, until all joints are water and air tight.

An equally reliable pressure test is made by using a force pump and a manometer.

For occasional inspections of old plumbing work, and in making sanitary examinations of houses the "peppermint" and the "smoke test" become useful. The peppermint test is thus described: "When called on to detect a leak in the soil pipe of a house, the plumber goes at once to the roof, if the soil pipe be carried above the roof; if not, he goes to uppermost water closet, and pours into one or the other something like an ounce of peppermint, and follows it up with enough water to insure its being carried the full length of the soil pipe. (The top of soil pipe should be closed, in order to prevent the oil from escaping into the outside air.) "Another man then traces the soil pipe from the bottom, throughout its course; knowing that if there is any crevice through which sewer gas can enter, the pungent odor of the volatile essential oil will be readily perceptible even in the presence of odors of a baser kind. Great care must be taken not to carry the peppermint about the house, otherwise the smell cannot be traced to the drains."

Captain Douglas Galton describes another test thus: "To test the drains the

fumes of ether or of sulphur may be used. If ether is poured down a soil pipe the fumes will be perceptible in the house at any leaks in the soil pipe or failures in the traps. Sulphur fumes may be applied by putting into an opening made in the lowest part of the drain an iron pan containing a few live coals, and throwing one or more handfuls of sulphur upon the coals, and closing up the opening to the drain with clay or otherwise. The fumes will soon be very perceptible at any leaks or rat holes in the soil pipe, drains or traps."

The connections between fixtures and the soil or waste pipes are made with lead pipe, which can easily be handled, and may be bent and cut to suit all possible positions, and requires but few joints. It is manufactured in long coils, of all sizes and of any desired thickness. In good plumbing work only heavy lead pipe should be used to prevent its being quickly destroyed by the corrosive action of sewer gas. It is desirable that lead pipe should be used as little as possible in *concealed* places, as it may be gnawed by rats or split by nails through carelessness of carpenters.

It is not uncommon to find vertical waste pipes of lead, as these are easily placed inside of a partition and covered with plaster. But this cannot be regarded as good practice; iron for waste pipes is decidedly to be preferred.

Vertical lines of lead pipe should be fastened to boards by soldering hard metal tacks to the pipe and screwing the flanges of the tacks to the board. Horizontal lines should be continuously supported on boards between joists. Lead pipes are mostly joined by what is called a "wiped joint." The end of one pipe is flanged out so as to form a cup, into which the other pipe, the end of which should previously be sharpened, is introduced. Hot solder is then applied to the joint, and wiped around it so as to form an oval lump.

Where lead pipes are joined to iron pipe, the connection should be effected by means of a brass ferrule of the same bore as the lead pipe, and soldered to it, wherever space allows, by a wiped joint. The ferrule is introduced into the hub of the iron pipe, and caulked tightly with a gasket of oakum and molten lead.

The size of lead waste pipes should be as small as is consistent with the office which they have to perform. Wastes for bath tubs or laundry trays should be sufficiently large to empty these vessels in a short time.

The following sizes of waste pipes for fixtures should be recommended:

For wash basins	1¼	inches diameter.
For wash basin overflows	1¼	" "
For bath wastes	1½	" "
For bath overflows	1¼	" "
For wash tub wastes	1½	" "
For kitchen sink wastes	1½	" "
For pantry sink wastes	1¼	" "
For slop sinks	1½ to 2	" "

Local conditions will, in some cases, demand a deviation from these sizes.

ON WEYRAUCH'S FORMULAS FOR THE STRENGTH OF MATERIALS.—By A. Brüll.—Admitting the value of Wöhler's experiments, it was best to retain the primitive limit of elasticity as the standard of working resistance. It had been shown by experiment that under certain conditions neither limit of elasticity nor breaking strength preserved their primitive values. But in working practice such conditions seldom existed, and the former might then safely be held to possess a definite and constant value. Wöhler had, in many instances, broken specimens of iron and steel by alternation of equal opposite stresses below the elastic limit; but the stress was very rapidly reapplied, though not with shock or absolute suddenness. It was well known that the minimum intensity of a suddenly applied load, required to produce a given elongation was half that of the corresponding statical stress, when the given elongation was below the elastic limit. From this it was inferred by Lippold, that the sudden application of stress below the limit of elasticity, but exceeding half its value, produced some permanent set, and at each repetition of the same stress a certain amount of work was spent in producing that result; rupture following when the total work so expended attained a sufficient value. The complex methods of calculation of Dr. Weyrauch, could not replace that based on the limit of elasticity until, for different qualities of material, prolonged experiment had furnished more definite values for the new coefficients.—*Résumé de la Société des Ingénieurs Civils, Paris.*

THE DIKES OF ISLE DE RÉ.

Translated from Annales des Ponts et Chausees for VAN NOSTRAND'S ENGINEERING MAGAZINE.

A GREAT portion of the Isle of Ré, especially the west-north-west part, is below high-weter mark and is protected from the sea by low dunes and by dykes whose total length is more than 9 kilometers (5.6 miles).

As these dykes preserve a territory occupied by a considerable population, they are regarded as works of great importance, and the continual care bestowed upon them is fully justified.

Before 1789 they were maintained by contributions levied upon those directly interested, and by the budget of the province of Aunis, for which the island was considered a sort of breakwater. The State also aided in the maintenance by a relief fund.

After the Revolution the State assumed entire control of the dikes, and they are now regarded as works of generl interest.

Formerly the outer slopes were covered with loose stones resting upon clay, but as this construction offered but poor resistance to the sea the breaks were numerous and were repaired only at considerable expense. Then the method of fascines and stakes was tried, but soon abandoned on account of the rapid decay. After this a method in imitation of the plan practised in Flanders was tried, and a slope of dry masonry was laid upon a bed of broken stone 16 to 20 inches in thickness. This construction held for a time, but when a breach was once made by waves in stormy weather it enlarged with frightful rapidity.

It was finally decided in 1846 to cover the slope with masonry laid in hydraulic cement with a total thickness of two feet; an outer course of one foot thickness being rough ashlar, and the under course of equal thickness being rubble. This system succeeded perfectly.

The slope of the masonry is for the most part 2 to 1. The inner face of the dike or levee has a slope of 1½ to 1. It is covered with clay and planted with Tamarisk which grows readily on the island. The dikes have a width at the top varying according to circumstances, but is generally two meters, and raised to a height of three meters above the highest tides. This height would be insufficient during great storms to prevent the waves from breaking over the work to the injury of neighboring plantations. The dike is therefore surmounted with a parapet two feet high (om. 6), so formed that with the outer slope the cross section is a parabola with a horizontal axis. By this construction a lower height of wall suffices to resist the waves.

The masonry generally rests on the limestone rock which underlies the whole surface of the island. When the rock is too low for this purpose, the work is made to rest on a tolerably firm substratum of earth which is found below the sand. It is rarely necessary to go deeper than three or four meters for this purpose.

It was at first thought necessary to protect the foot of the wall, where it was not founded on rock, by a system of sheet piling. It was not however required. In calm weather there are no waves to cause damage, and in stormy weather the retreating wave sliding down the masonry slope meets another wave so that the stonework receives the shock, and the sand at the foot is not disturbed.

The dikes of Petit Pres (Fig. 1) and of Maison Neuve (Fig. 2) represent the different forms employed in Isle de Ré.

The cost varies with the price of material, but averages for the type of Fig. 1 100 francs per meter or 18 dollars per lineal yard, and for the other variety 150 francs per meter or 27 dollars per yard. This estimate does not include cost of land, which is generally government property.

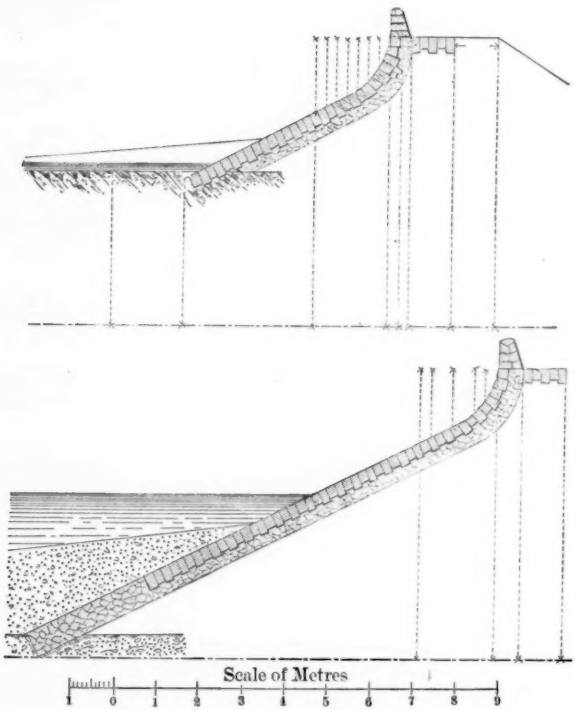
The products of the sea are not of much benefit to the inhabitants of the island, but it is nevertheless necessary to construct at convenient distances approaches to the shore, which may be used as roadways for the transportation of fish or of such materials as are used as

fertilizers of the land. These roadways increase somewhat the sort of the dike.

Along the greater part of the coast of the island there is a body of sand carried along by the littoral currents. The plan of causing a deposit by means of groynes was tried, but soon abandoned. Sand was deposited on the up-stream side, but the shore was eroded to a coresponding extent on the other. Only where the shore was naturally very solid could the plan be profitably adopted.

These dikes are, as already stated, 9 kilometers in length, and being of vital importance to the country, they are the object of continuous and careful surveillance.

A brigade of skilful cantonniers are in constant attendance to repair at once any breach in the wall, and who are required especially to act with promptness in mending the breaks occasioned by storms. It is necessary to be carefully guarded with solid materials at all dangerous points. The present type of dike has successfully resisted the sea for twenty years. The older form is occasionally broken through in places never before disturbed. In restoring such portions the modern type is always made to replace the ancient.



The annual cost of the maintenance of the dikes of the Isle de Ré is 25,000 to 26,000 francs ($5,000 to $5,200). This amount would quite cover all sorts of repairs and maintenance if they were throughout of modern construction.

THE electrical perturbations were so frequent on the French lines from April 16 to 20, that measures had to be taken by the Minister of Postal Telegraphy to meet this contingency. The electrical equilibrium was restored on the 21st. These electrical perturbations were noticed on the telegraphic lines of Germany, Belgium, and Italy, and of England, according to the notice which was published by the French Administration in the official paper of the Government.—*Nature.*

THE CONSERVANCY OF RIVERS: THE EASTERN MIDLAND DISTRICT OF ENGLAND.

By WILLIAM HENRY WHEELER, M. Inst. C.E.

Proceedings of the Institution of Civil Engineers.

THE conservancy of the rivers of this country is a question continually growing in importance. It is one which must before long be dealt with by Parliament, and legislation effected which will necessitate considerable engineering works for putting the arterial drainage of the country on a more satisfactory footing. The frequent recurrence of floods, and the immense damage caused by them, cannot be allowed to go on without a remedy being sought.

Much valuable information as to the best method of forming a proper organization for the management of rivers has been elicited by Parliamentary Committees and public discussions on the subject, and individual engineering opinions have been given as to the way in which Floods Prevention Works should be carried out. No opportunity, however, has yet been afforded for a general expression of engineering opinion and discussion of the principles on which the regulation of rivers should be conducted. Such a discussion will be highly valuable, not only to those members of the Institution who may hereafter be called upon to carry out these works, but also, as a basis for the guidance of those on whom lie the responsbility of deciding the best course to pursue, and of levying the money to pay for the works. From want of a clear perception of the principles which ought to guide all works for the improvement of rivers great mistakes have been made, enormous sums of money have been wasted, and taxes levied from which little or no benefit has been derived.

The circumstances of river basins in this country are so various in character, owing to geological and economical causes, that it is not possible to lay down any method of dealing with all rivers alike. Still there are certain general principles that should prevail, and which should be borne steadily in mind in designing improvements, whether of a local or a general character. In the following paper an endeavor will be made to show what in the author's opinion these principles are, and to illustrate them by facts relating to one particular class of rivers.

The rivers here dealt with are those draining the Eastern Midland portion of England, and are typical of the drainage systems of flat districts, of permeable strata discharging into sandy estuaries, with a small rainfall, free from mountain torrents, and rapid discharges of water met with in the watersheds of volcanic districts. The industry pursued on their banks being mainly of an agricultural character no complication arises from the pollution by manufactories.

Large sums of money have been expended on these rivers, for which some of the lands draining by them are heavily taxed. Yet owing to the piecemeal way in which this has been done, these river basins are still subject to most disastrous floods. If the same amount of money had been judiciously expended on a comprehensive plan embracing the whole river system, and the cost fairly spread over the lands benefited, these rivers would now be in a comparatively efficient state, and competent to discharge the heaviest floods without any undue burden being imposed on the land.

The Eastern Midlands lying between the Trent, the Severn, and the Thames, are drained by four rivers, the Witham, the Welland, the Nene, and the Ouse, which discharge into the upper end of a large indent or bay on the east coast known as "The Wash." There are other small rivers draining the district lying between the watersheds of the Ouse and the Thames, which discharge at various points along the coast, but these it is not intended to deal with. The area drained by the four rivers is about 5,719 square miles; their total length about 416 miles, and with the tributaries

872 miles. The number of square miles to a mile in length of the main stream is 12.74, or 8,155 acres for the whole watershed. Including the affluents there are about 4,015 acres to a mile of river.

These rivers drain portions of the counties of Lincoln, Norfolk, Northampton, Cambridge, Huntingdon, Rutland, Bedford, and Buckingham. The principal towns within the watershed are Lincoln, Boston, Grantham, Spalding, Wisbech, Peterborough, Northampton, Lynn, Cambridge, Ely, Bedford, and Dunstable. With the exception of Northampton, where shoemaking is carried on to a large extent, and Bedford and Dunstable, where the strawplaiting industry is chiefly located, these towns are mostly agricultural centers, and are markets for the disposal of the produce grown on the lands around. The businesses carried on are almost entirely those for the supply of agricultural machinery, for the manufacture of the produce for market, or of oil cake or other food for the stock, and of artificial manures for the land. The rainfall of the district is small, ranging from 17.39 inches in the driest seasons to 34.48 inches in the wettest; the average being 26.05 inches.

The country generally is flat, and the elevation at the source of the rivers is only about 300 feet above the level of the sea. The geological formation is Kimmeridge and Oxford clays, Oolites with small deposits of Lower Greensand, Chalk and Glacial drift. The lower or fen districts are alluvium and peat. The sources of the four rivers are not more than about 30 miles apart, the water producing the streams breaking out from the Oolites near the extreme northeastern boundary of the watershed of the Severn. The lower part of the watershed, comprising about 668,241 acres, is a plain, known as "The Fens," now a tract of valuable agricultural land, but formerly a morass, which in winter, with the exception of a few elevated spots, was little better than a lake, but in summer afforded valuable pasturage for the cattle of the occupiers of the adjoining high land. After the introduction of monastic life into this country, settlements took place in the Fens by some of the religious orders. The abbots and priors began gradually to improve portions of the fen, but no systematic attempt at reclamation was made until the seventeenth century, when certain speculators or "adventurers" undertook to drain and improve the fens in return for a share of the lands. The most successful of these was the Duke of Bedford, who reclaimed a large tract of land in Cambridge and Norfolk, known as "The Bedford Level," much of which is owned by the successors of the original "adventurer."

The adventurers called to their assistance Vermuyden, a Dutch engineer, who designed his works of reclamation on a plan similar to plans adopted in Holland. Losing sight of the greater range of the tides in the estuary than on the coast of his own country, he took no advantage of the gain to be obtained by discharging the drainage direct into the estuary, where low water ebbs out lower than the North Sea, and thus securing a natural outfall for the water. The outfalls were neglected, embankments were made along the main rivers, and long arterial cuts through the lands to be reclaimed, with sluices at the end to keep out the tidal waters. Under this system the lower part of these river-basins became split up into a number of districts or levels, each level dealing with its own drainage irrespective of its neighbors. The aggregate amount of money thus spent in the reclamation works was far greater than it would have been had all contributed to the improvement of the common outfall. Conflicting interests were created which have since caused enormous sums to be spent in litigation, and have prevented that common action for the improvement of the rivers which is generally admitted to be necessary, and adding greatly to the difficulties of the application of any system of river conservancy.

As the original works failed to attain the purpose for which they were intended, fresh cuts were made. In many instances the course of some rivers was entirely diverted. Long straight cuts were made to supersede the winding course of some natural rivers, shortening considerably the distance the water had to travel, and accelerating their discharge. In these new rivers the flood-banks were set in some cases as much as a mile apart, the river channel occupying a space in the center sufficient

only for the ordinary discharge of water. In floods the water overflowed the ordinary banks, and spread over these "Wash lands." The country below being at that time almost entirely open marsh, the outfalls were thus capable of receiving the flood-water, and the washes being unobstructed, the floods passed away without doing any damage to the land, which was then all under grass.

The marsh lands below these washes have subsequently been reclaimed, and the outfalls otherwise choked and impeded, and the washes have long ceased to answer the purpose for which they were originally intended. Where they have not been encroached upon by being embanked from the rivers, they now in times of flood become vast lakes, which fill with water on the overflowing of the rivers, sometimes to a depth of 6 feet, the water remaining on them for several weeks together, presenting the appearance of an inland sea. The proprietors having become possessors of portions of these washes at high prices, have sought to recoup themselves by endeavoring to grow crops of hay, and in many instances by turning the fields into arable land. During the last few years, owing to the continuous floods, crops have been washed away, and the land rendered of little value. The miasma arising from this land, when at length it begins to dry, after several weeks' submergence, is prejudicial to health. Thus what were intended by the engineers who designed these wash lands as flood regulators, have, by the want of a general system of control, become a nuisance.

The existence of these washes, the large area they cover, and the above facts, are sufficient answers to those theorists who are in the habit of advocating the formation of reservoirs to regulate the streams and prevent floods. Here, on rivers draining comparatively a flat country, are occasional reservoirs of 3,000 and 5,000 acres, which yet have scarcely any effect in preventing most severe floods on the lands above them. Taking an average depth of water of 4 feet over the whole of the wash lands, those on the Nene would only provide for a rainfall of 0.297 inch over the watershed draining above them, and those on the Welland of 0.48 inch.

THE WITHAM.

The Witham rises near Thistleton and South Witham, a few miles north of Stamford, at an elevation of 339 feet above the sea. It is about 89 miles in length, has five tributaries, the Brant, the Till, the Langworth, the Bane, and the Sleaford River, their united length being about 98 miles. The area of the basin drained is 1,063 square miles, of which 196,686 acres are fen lands. The number of acres to 1 mile in length of the river and its tributaries is 3,635. The tidal flow only extends 8 miles, the tide being arrested at Boston by a sluice placed across the river, having self-acting doors, which close against the tide and open on its receding. The tide flows from two to three hours, and at spring tides there is a navigable depth at the present time of about 16 feet. Mean high water on an average of four years (1869–72) rose 18.92 feet above the Black Sluice sill at Boston, or 10.22 feet above ordnance datum; spring tides, 22.02 feet; neaps, 15.36 feet. A spring tide which rose 23 feet 4 inches in Clayhole, rose 13 feet 2 inches at Boston; and a neap tide, which ranged 9 feet 2 inches in Clayhole, ranged 6 feet at Boston. By the works now being carried on under the Witham Outfall Act of 1880, it is expected to give a navigable depth of 22 feet at the proposed entrance to the new docks at Boston.

Between Boston and the lock at Bardney, a distance of 20 miles, water is maintained for purposes of navigation at a uniform depth of 9 feet. The Commissioners have now, under the Act of 1881, obtained power to reduce this when necessary. In floods the regulating doors at the Grand Sluice at Boston are withdrawn, and the water allowed to flow without interruption. The sluice has four openings of 16 feet each, and the depth of water on the sill at ordinary floods is about 10 feet, rising as high as 14 feet in extreme floods. The fall in the surface of the water in floods between Bardney and Boston is from 3 to 5 inches per mile, and between Boston and the sea 25 inches per mile. The waterway of the river about 2 miles below Boston is 200 feet at low water. With 10 feet of water the area is 2,000 square feet.

The area drained through this part of the channel is 650,392 acres, thus giving 325 acres to every square foot of waterway. The waterway of the Grand Sluice is 66 feet, and with a depth of 10 feet on the sill it has an area of 660 feet. The river above was originally excavated so as to give a mean waterway corresponding with that of the sluice. The number of acres draining through the sluice is about 448,835, being 680 acres to a square foot. The area of the river at Boston at ordinary low water is 156 square feet, and at high water of spring tides 2,286 square feet, a proportion of 1 to 14.6. But in comparing this with the other rivers, it must be borne in mind that the section is taken only 7 miles from the estuary, the tidal flow being arrested at the Grand Sluice, 1 mile further up the river.

The river has been considerably altered below the City of Lincoln, from which place it is mostly artificial. About the middle of the last century the banks on both sides of the river from Boston to Lincoln were raised and strengthened, the greatest of the bends removed by new straight cuts, and the channel generally deepened, widened, and improved. The Grand Sluice was erected for preventing the tide flowing into the upper reach of the river. These works were completed in 1766, at a cost of about £53,650. In 1811 a further amount of £30,000 was spent in this portion of the river. Additional works have been carried out under an Act obtained in 1865, for deepening and removing obstructions from the channel, and strengthening and raising the banks. The cost was about £50,000. The navigation authorities have expended, during the last fifty years, about £60,000 in straightening and training the tidal portion of the river below Boston. Under an Act obtained in 1880, works are now being carried out for making a new outfall by a cut 2½ miles in length, by which the distance will be shortened 1½ mile, and the shifting sands at the mouth of the river avoided. It is expected that this will give relief of at least 3 feet in the low-water mark at the drainage sluices.

The cost of the works executed up to the present time is upwards of £300,000, and the works for the outfall are estimated to cost £120,000 more. Beyond this a large sum has been spent on works for improving the river by the owners of the upper navigation. The cost has been met by taxes on the low lands and by dues on the shipping. The taxes on the fen lands for river works vary from 1s. to 5s. 6d. an acre, in addition to what has to be paid for works of interior improvement, which on some of the fens brings the amount of drainage taxation up to 16s. per acre. This amount extends over a length of 35 miles of the lower part of the river, or only about one-half of its course. Further expense has been incurred in straightening and improving the upper reaches, by which the water is discharged more rapidly into the lower part, but the landowners contribute nothing to the works below Lincoln. Notwithstanding the improvements, the river is incapable of discharging the water as quickly as it is poured into it, owing to the defective outfall at the sea, to the obstruction caused by the sluice at Boston, the weirs at Lincoln, and the inadequacy of the channel between those places, and consequently the floods on this river have been increasingly frequent and disastrous. The lower part of the city of Lincoln has been several times under water, the houses for a time being rendered uninhabitable and the large engineering works stopped. In the winter of 1876, when several of the interior banks were broken, 40,000 acres of land were under water, people were driven from their houses, and cropping was lost to the estimated value of £100,000. In 1878 and 1879 there were very heavy floods; and in the autumn of 1880 a large tract of land was again submerged; the corn stacks were standing several feet in water, and sheaves of corn which had not been carried away were floating about in the fields. Not only were the farmers injured, but much valuable food was destroyed.

THE WELLAND.

The Welland rises in a gentle range of hills between Lutterworth and Market Harborough, near the source of the Ise, a tributary of the Nene. It is about 72 miles long, has three tributaries, together 65 miles long, and drains about 707

square miles, of which 76,854 acres are fen land. The number of acres to 1 mile in length of the river and its principal tributaries is 3,302.

The Welland has a tidal course of 20 miles; extreme tides reach as far as Crowland. A spring tide which rose 23 feet 4 inches at Clayhole rose 12 feet 2 inches at Fosdyke bridge, 8 miles from the estuary, and 4 feet at Spalding, 15 miles from the estuary. When the river is thoroughly scoured out to its full depth the rise at spring tides is 8 feet, giving 10 feet at high water of spring tides. The range of a neap tide, which was 9 feet 2 inches at Clayhole, was 5 feet 5 inches at Fosdyke, but the tide did not reach Spalding.

The mean inclination of the surface of the water between Spalding and Clayhole at ordinary low water is 14 inches per mile. During floods, in the trained portion of the channel below Fosdyke bridge, the inclination is 9 inches per mile, and between Fosdyke and Spalding 2 feet per mile. In large floods the average inclination from Spalding to low water of spring tides in the estuary, 15 miles, is 21 inches per mile. Owing to the want of prolongation of the trained channel, the fall from Fosdyke bridge to low water in Clayhole, 8 miles, averages about 18 inches per mile, due to the great fall between the end of the trained work and Clayhole.

The average waterway of the river at Spalding is about 40 feet, and the area in floods 400 square feet. The drainage area discharging there 300,000 acres, giving 750 acres to a square foot. The mean width of the trained channel below Fosdyke is 120 feet; the area of the waterway with 10 feet depth of water is 1,200 square feet. The drainage area discharging through this channel is about 452,480 acres, or 377 acres to a square foot.

The area at Spalding at low water is about 73 square feet, and at high water spring tides 485 square feet, a proportion of 6.65 to 1.

The Welland retains its ancient course more nearly than any of the other rivers, yet it has been considerably altered. The river was made navigable from Stamford to the sea by improvements in the channel of the river, straightening the same by new cuts, and by the erection of locks, &c., the first lock on the river being about 13 miles above Spalding. Subsequently the adventurers of Deeping fen, in order to obtain a better outfall for their drainage, widened and deepened the river below Spalding. In the year 1801 a new cut was made from the reservoir 8 miles below Spalding, and the open marshes above Fosdyke were enclosed. About forty-five years ago the work of training the river by fascine work through the shifting sands below Fosdyke bridge was commenced and continued for a length of 3 miles 30 chains. This training had the effect of lowering the low-water level at Fosdyke bridge 7 feet. The whole of these works, so far as they relate to the improvement of the river as the outfall of the drainage of the country, were paid for by the Fen land in the low level of the river basin, assisted by dues levied on the shipping using the artificial channels.

The arterial drainage of this district is still in a very defective condition, the channel not being sufficiently adapted to carry off the rainfall as rapidly as it is collected in the river. The banks which protect the fens are constantly being broken, owing to the channel being overfull and the fens flooded. The repeated floods of the last few years have done an immense amount of damage by submerging the land and destroying the crops. In July 1880, in addition to thousands of acres of land which were submerged, the whole of the lower part of the town of Stamford was flooded, as were also the villages of Market Deeping, Elton, Maxey, and others on the course of the river, the water rising to a height of 3 and 4 feet in some of the houses. Again, in the autumn of the same year, a flood, almost as extensive and if anything more disastrous in its results, occurred. Although floods so calamitous are exceptional, yet their frequency and the large area of land thrown out of cultivation, are sufficient to demand that such alterations should be made in the river, as the main outfall of the drainage of the district, as to render it efficient for its purpose.

THE NENE.

The Nene rises in two springs at Daventry, and owing to its windings, although in a direct course the distance is

only 60 miles, the length of the river is 99 miles. It has three tributaries: the Ise, the Harper, and Willow Brook, their united length being 52 miles.

The Nene has a drainage area of about 1,055 square miles. The number of acres to 1 mile in length of the river and its tributaries is 4,474.

The tidal flow is 34 miles, at spring tides, reaching Northey Gravel within 2¾ miles of Peterborough, and at extreme tides even as far as Peterborough. The tide flows three and a half hours at Sutton bridge, 7 miles from the estuary, and two and three quarter hours at Wisbech, 15 miles from the estuary. A spring tide, which rose 23 feet 3 inches in the estuary, rose 20 feet 6 inches at Sutton bridge, and 15 feet 2 inches at Wisbech. A neap tide of which the range was 9 feet 1 inch in the estuary, ranged 8 feet 5 inches at Wisbech. The navigable depth of water at Wisbech is about 22 feet at high water spring tides, and 3 feet at low water. From observations made by Sir John Coode, M. Inst. C.E., it appears that, owing to the tide being throttled by the contracted form of the lower part of the channel, it has not free ingress and egress, and does not reach the limit of its flow until some time after the ebb has commenced at the lower end. Thus the particular tide observed ebbed three and a quarter hours at the lower end of the trained portion of the channel before it had reached the "Dog in the Doublet," 25 miles above, and then continued flowing there for forty-five minutes. The water rose 6 feet at the upper end, while it fell 6 feet 11 inches at the lower end. Thus there are two strong currents in the river running simultaneously in opposite directions, the ebb towards the sea and the flow towards Peterborough. High water spring tides is 7 feet lower at Peterborough than at the outfall at Stone Ends, and at neap tides it is 8 inches lower at Cross Guns, 24 miles from the outfall.

The mean inclination of the surface of the water at low water from Peterborough to the sea is at the rate of 5.63 inches per mile. This rate varies considerably along the different sections, the minimum being 2 inches per mile along the lower reach, and the maximum at the Horse Shoe bend at Wisbech 14½ inches per mile. In severe floods the inclination from the South Holland sluice above Sutton bridge to low water at spring tides in the estuary, 8¼ miles, is at the rate of 10½ inches per mile. Through Wisbech, in great floods, there is a fall of 3 feet in less than a mile.

The mean waterway of the river in the upper reach, a short distance above Wisbech, is 50 feet, giving an area with 10 feet depth of water of 500 square feet. The area of land draining through this part of the river is about 564,700 acres, or 1,129½ acres to a square foot. In the lower reach, between the stone banks of the trained channel, the waterway is about 220 feet, and with a depth of 10 feet the river has an area of 2,200 square feet. The area of land drained is about 675,200 acres, being 307 acres to a foot. Taking the area above Wisbech at ordinary low water at 240 square feet, and at high water of spring tides 1,595 square feet, the proportion of tidal to fresh water for the ordinary flow is 6.65 to 1.

The Nene is navigable from Northampton; it enters the fens at Peterborough, and then divides into two branches, one branch, the old river, joins the Ouse by a branch from Stanground sluice. The main stream runs by Smith's Leam through the wash lands and Wisbech to the sea. The Nene has been more altered by various works than any other river. From Peterborough to the sea it is nearly a new river. Bishop Morton in 1478–86 first commenced the alterations, diverting the river from its original course by a new cut from Peterborough to Wisbech, about 11 miles in length, which shortened the course of the water 7 miles. In 1726 the present channel of the river between Peterborough and Guyhirne was made, its course being parallel with Morton's Leam. The banks are about ½ mile apart, leaving 3,500 acres of low-lying meadow land or "washes" At Guyhirne, 6 miles above Wisbech, these banks come together and are close upon the river. From the Horse Shoe bend towards the sea below Wisbech a channel was cut by King Charles. In 1773 a new cut was made 1½ mile in length 5 miles from Wisbech, since known as "Kinderley cut;" and between 1827 and 1832 this was continued by the Woodhouse, or "Pauper's cut," so called from a number

of paupers having been employed on the works. About fifty years ago the improvement of the river below these cuts was continued by excavating and scouring a new channel through the Cross Key washes from Gunthorpe sluice to Crab's Hole, a distance of 5 miles, with further training banks through the sands about 1¼ mile in length. A large tract of land was at the same time reclaimed. The new outfall lowered the low water at the North-Level sluice 10 feet. In 1813, before the last improvement was made, the fall in the surface of the river was at the rate of 3 feet per mile. Afterwards it was only 3 inches in the mile. In 1852 further powers were obtained for improving the river between Peterborough and the sea, and after an expenditure of £200,000 the works were discontinued without any material improvement having been effected. The alteration in the channel of the river greatly augmented the range of the tides. In 1769, according to a report of Golborne, spring tides only rose 4 feet at Wisbech, and neap tides did not reach the town; after the new channel was made they rose from 15 to 16 feet.

Within the last century the amount spent on the improvement of the main channel of the Nene has been upwards of £450,000, about one-fourth of which sum was raised on the navigation dues, to meet which all ships entering the port are subject to a charge of 1s. 0½d. per ton-register, and the remainder by the fen land. The taxes on the land to meet this outlay reach in some cases 15s. an acre, and yet the land is occasionally flooded. The river is in a most unsatisfactory condition, thousands of acres of land along the valley being sometimes inundated, and even the streets of Peterborough flooded and people driven from their houses, while the whole arterial drainage system suffers from its defective condition.

THE OUSE.

The Ouse rises at an elevation of 300 feet above the sea in numerous springs; these escape from the Oolite escarpment at its junction with the Lias Clay above the valley of the Cherwell, between the Ouse and the Thames, and within 4 miles of one of the sources of the Nene. The head of the main branch is about 87 miles from the sea, but owing to the tortuous course of the river the length of the channel is 156 miles. It has ten tributaries, their united length being 241 miles. The drainage area is 2,894 square miles. The number of acres to 1 mile in length of river and tributaries is 4,672. The river for the last 50 miles of its course runs through a flat low-lying district, and has been embanked from St. Ives downwards. Spring tides flow a considerable distance up the Hundred-Foot river, or nearly to Earith, 20 miles beyond Denver sluice, giving a tidal course of 40 miles.

The average rise of a spring tide at the Free bridge above Lynn, as taken from the records observed there over a period of seven years (1869–75) was 18.51 feet above zero, which is about 1.31 foot above low water of spring tides. The highest tide observed during that period was 22 feet 6 inches, an average neap tide was 12.04 feet, and the mean of all tides 15.54 feet, or 10.59 feet above ordnance datum. A spring tide, which rose 23 feet 3 inches above low water in Lynn Roads, rose 22 feet 6 inches at Lynn; and a neap tide, which ranged 9 feet 1 inch in the estuary, ranged 9 feet 5 inches at Lynn. The tide flows for about 5 hours at Lynn.

The ordinary low-water inclination of the surface of the water along the Eau Brink cut is about 3 inches per mile. In large floods the mean inclination from Denver sluice to low water in the estuary, a distance of 19 miles, is at the rate of 9 inches per mile. From Denver to Lynn the surface inclination is 12 inches, and from Lynn to the estuary 8 inches.

The area of the waterway of the river above Earith is very irregular. That of the channel near Earith is only 243 square feet, while 7 miles further up the river, near St. Ives, has a sectional area of 672 square feet. At Over Court Ferry the area is 492 square feet. The area of the outlets for flood-water above Earith was found by Mr. Abernethy, President Inst. C.E., in 1875 to be 4,233 square feet, while below the Seven-hole sluice at Earith it was only 2,058 square feet. The shuttles at the Seven-hole sluice are not lifted till the flood-waters have risen 4 feet 6 inches above the level of the wash lands, or until a large part of

the country is flooded. The fall in floods from the upper to the lower side of this sluice is 2 feet, caused by its restricted size as an outlet for the large area which has to drain through the sluice. In the Eau Brink cut the area in floods is about 2,620 square feet; and the drainage area being 1,852,160 acres, gives about 707 acres to a foot. In the Marsh cut the dimensions of the cut, originally set out with slopes 4 to 1, have increased by the washing away of the banks from 265 feet at the bottom to an average of 425 feet, and from 500 feet at the top to an average of 594 feet. The depth originally was 10 feet 4 inches, and now varies from 10 feet to 19 feet, averaging 12 feet 8 inches. The channel below the Marsh cut, where it is confined by guide-walls of stones and fascines, is 400 feet wide, and, taking the depth at 19 feet, gives 463 acres to a foot of sectional area of waterway.

The section of the Eau Brink cut has also become very irregular since its first formation. From a number of measurements in 1862 it was found that the sectional area at low water in some places was double that in others, and the depth at low water varied from 17 feet 3 inches to 2 feet 9 inches. The mean of forty-three measurements gave the area at ordinary low water as 1,824 square feet, and at high water of spring tides 9,421 square feet, a proportion of 5.16 to 1.

The average low-water level of ten years, 1844–53, previous to the completion of the Marsh Cut, was 2 feet $5\frac{1}{2}$ inches above the datum at the Free bridge, and for ten years after the opening of the cut, 1866–75, $9\frac{1}{4}$ inches below, showing an average gain of 3 feet $2\frac{3}{4}$ inches. The extreme low water varies from 3 feet 6 inches above datum to 3 feet 6 inches below, or a range of 7 feet. The average low-water level of spring tides at the Free bridge is now about 1 foot $3\frac{3}{4}$ inches below datum, or 3 feet 8 inches above low-water spring tides in the estuary; and during neap tides $2\frac{3}{4}$ inches above datum, or 5 feet 3 inches above low water.

The Ouse stands first of all the Fen rivers in the large amount of money which has been expended in its improvement. Without taking account of what was done by the early adventurers, upwards of £800,000 have been raised and expended in making new cuts, and otherwise improving that portion of the river which passes through the Fen land. The benefit of these improvements has been enormous, the low-water level having been depressed 12 feet.

Vermuyden began the alterations in this river in 1638 by making a new cut 21 miles long and 70 feet wide, called the Old Bedford river, from Earith, where the river enters the fen jurisdiction, to Denver sluice. In 1652 the New Bedford, or Hundred-foot river, was made parallel with the other; and banks were raised on the north side of the old Bedford river and the south side of the new river, leaving an area of 5,000 acres of wash lands between. By this cut the course of the river was shortened 10 miles; and the old course of the river being maintained, there were three channels for the river. In 1748 Denver sluice was erected, by which the tidal flow was stopped from going up the old river course, but was still allowed a free run up the Hundred-Foot river. Subsequently the Hermitage, or Seven-hole luice, was erected at Earith, and all thes water coming from the basin of the Ouse above this, extending to 756,000 acres was discharged by the new river, while the old Bedford river and the wash lands afforded receptacles for the waters in extreme floods. By an Act passed in 1812 the owners were allowed partly to embank the washes, and they have since been gradually encroached upon, their use as flood-regulators being otherwise destroyed.

The Eau Brink cut was originally projected by Kinderley in 1720, and the Act was obtained in 1795; but it was not completed until 1821. The original estimate was £39,985; the ultimate cost, £600,000. The length of the cut is $2\frac{1}{2}$ miles, the old course of the river being 5 miles. The effect of the cut was to lower the low water 6 feet at Denver sluice, and 8 to 9 feet at Eau Brink, where the new cut joined the old river. In 1853 the Norfolk Estuary Company made a new cut through the marshes below Lynn 2 miles in length, and continued the channel by training through the Vinegar middle sands for a distance of about a mile. The cost of this work was upwards of £200,000, towards which

the drainage and the navigation contributed £110,000. This cut shortened the course of the river, and depressed the low-water level 3 feet at Lynn. Since the opening of the Marsh cut the river has been further improved by dredging away a large clay bar or shoal lying between the Eau Brink cut and the Marsh cut.

INLAND NAVIGATION.

The present condition of the inland navigation seriously affects these rivers, and is one chief cause of their incapacity for carrying away flood-waters. Owing to the position of the Wash with reference to the Netherlands and the Continent, Lynn and Boston were once prominent ports, ranking only second to London and Bristol; and although a great portion of this trade was diverted by the opening up of Hull and other ports on the east coast, yet up to the time of the construction of railways there was a large export trade of wheat and agricultural products, and an import of coals and other goods which were distributed throughout the midland part of England by these rivers. Water carriage was almost the only means of conveying heavy products into the country, and of exporting the corn and wool; as this traffic increased, the rivers, where they became shallow, were canalized and made navigable by locks or staunches. Thus Bedford by the Ouse, Northampton by the Nene, Stamford by the Welland, and Lincoln by the Witham, with other smaller towns, were placed in communication with the sea.

So long as these navigations were maintained in order, the shoals cleaned out as they accumulated, the locks and staunches preserved in efficient condition, and the weeds cut or kept down by the traffic of the boats, the rivers even in their artificial state of canalization were capable of discharging the flood-waters; but since railways have diverted the traffic from these inland rivers, navigation has ceased, the works have gone to ruin for want of funds to maintain them, and shoals and weeds choke the channels. The rivers have become in a far worse condition to discharge the drainage of the country than when left in their natural state, and constant floods are the consequence. The proprietors of the navigations, who have suffered greatly by the loss of the dues, although unable to fulfil the duties belonging to a proper maintenance of the streams, still cling to the remnant of traffic left. For this they adhere to their rights as to the holding-up of the water, without having the means to adapt the rivers to the modern requirement of drainage by enlarging the capacity of the weirs, so as in times of flood to discharge waters sent down at a much greater rate than formerly.

On the Witham, for a distance of 30 miles between Boston and Lincoln, the river is practically a canal. The tide is stopped by a sluice at Boston, and a weir and locks had to be constructed at Bardney and Lincoln. The inland water is held up to a constant height on the sill of this sluice by penstocks, for the purposes of the navigation. The navigation having been taken over by the Great Northern Railway Company, the works are maintained in efficient condition, but the obligation imposed by the original Act of holding up the water seriously affects the drainage. The river Slea, from Sleaford to the Witham, was made into a canal in 1792. The navigation on this river having almost entirely ceased, the company was dissolved by an Act recently obtained. The Bane, another affluent of the Witham, was also canalized forming a navigation from the Witham to the town of Horncastle; but the dues obtained are insufficient to maintain the works in proper order.

On the Nene, which is canalized from Peterborough to Northampton, the navigation is reduced to a few barges. The constant floods on this river are ascribed in a great measure to the defective condition of the works. The proprietors of the navigation, on whom was cast the duty of maintaining the river, no longer have the funds, and there is nobody to take their place. The same thing has occurred on the Ouse between Earith and Bedford.

On some of the affluents of these rivers, which under legislative powers granted last century had been converted into "navigations," the proprietors have obtained Acts of Parliament relieving them of their rights and liabilities, and there is now no jurisdiction over these rivers, or anybody responsible for removing shoals or cutting weeds. The beds of

these streams have consequently grown shallow, and the rivers are no longer capable of acting as efficient arterial drains. Thus on the Ivel, an affluent of the Ouse, the navigation trust created in the reign of George II., was abolished in 1876. The river is said to have since diminished one-half in width and one-half in depth, and the bottom is being gradually raised above the level of the land. In like manner the Lark, another canalized affluent, has almost entirely silted up since the navigation of the river ceased. The Ouse itself above Earith is obstructed by numerous shoals, and an enormous growth of weeds. These were originally kept down by the constant passage of the vessels, and the shoals were removed by the trustees of the navigation.

It is no doubt a great advantage to the water supply, and also for the water power of the country through which these rivers pass, and conducive to the economical conveyance of gravel, stone, lime, manures, and other heavy materials, where time is of no great consequence, that the locks, weirs, and works should not be abandoned, and the rivers restored to their natural state; but it is desirable that these works should be placed under a jurisdiction interested in and having control over the drainage, and that by the enlargement and improvement of the weirs and other works the rivers should be placed in a state of efficiency.

CAUSE OF FLOODS.

From the improved system of drainage now pursued, necessitated by the higher cultivation of the land, the rain is more rapidly discharged into the rivers. The water is no longer suffered to fill the land like a sponge, and pass off either by evaporation or slow percolation through the subsoil, but rapidly soaks through the soil broken up and disintegrated by steam ploughing and deep cultivation, and as soon as the substratum is saturated to the level of the drain-pipes, the rain-water is carried to the ditches. Efficient pipe drainage necessitates clean ditches, and the straightening and improving of all arterial drains and minor watercourses. Thus every impediment is removed from the free flow of the water to the river. Large tracts of water known as meres, which formerly acted as reservoirs, have been drained; woods and plantations which absorbed and held the rainfall have been stubbed up. Villages and towns are drained, and everywhere, whether in town or country, every effort is made to prevent stagnation, and speedily to void the water. An increase in the rainfall has also no doubt contributed to the increase of floods. On examining the statistics of rainfall kept at Boston for the past fifty years, it appears that there has been a considerable increase in the annual rainfall during the last few years, and especially during the last five. The average annual rainfall of the last five years has been 29.04 inches, or a greater quantity than previously recorded during a like period, and 5.62 inches above the average of the last fifty years. The next wettest period was 1846–50, when the average annual fall was 4.22 inches less than during the last five years. Taking ten-year periods, the average annual rainfall of the last ten years has been 4.34 inches greater than of the previous ten years, and 4.78 inches more than the ten years 1851–60, and 1.83 inch over 1841–50. Taking twenty-year periods, the last twenty years is 1.14 inch in excess of the previous twenty years and 4.11 inches in excess of the previous fourteen years. The largest increase has been in the months of September, February, and December, and the least in July and October. During the last few years September has had the greatest fall, and March the least.

Meantime no provision has been made to meet this more rapid discharge. In the upper reaches of the rivers no adequate jurisdiction exists to prevent obstructions, to compel the maintenance of works, or to levy taxes for carrying out improvements. In the lower reaches the works have been done in sections, and without reference to the general drainage-system of the rivers, and have been for the benefit of, and are paid by, the low lands, the owners of which of course are opposed to any improvements which will bring the upland waters on to them more rapidly. In fact, so jealous are the managers of the lower reaches of the river, that powers have been obtained

enabling them to regulate the quantity arriving from the upper reaches. On the Ouse at Earith a sluice regulates the flow of water from above, in which the openings are not only too contracted to allow the flood-waters to pass freely through, but the shuttles are not lifted until the water has risen to more than flood-height on the lands above. In the Witham, at Lincoln, the quantity of the discharge is regulated by a weir, which is inadequate in times of flood, but any increase in the size of which is prevented by the Commissioners having the control of the drainage below, the consequence being that the lower part of the city and upwards of 15,000 acres of land above this weir are frequently flooded.

The openings of the bridges across the rivers, most of which were built before the conditions of drainage were altered, are many of them totally inadequate to the discharge of the waters, and great discrepancies exist in the area of the waterways. Thus Mr. Abernethy states in his report on the Ouse that the bridges over the Hundred-Foot river have only half the area of the waterway of those at St. Ives 12 or 13 miles higher up.

The growth of weeds, and the increase in the cesses or banks of the rivers which have gradually encroached on the waterway, form another serious and increasing obstruction. Owing to the careless way in which the weeds are cut in some of the rivers, they are allowed to float down the stream, settling in the shallow places where sand and alluvium collect, in time forming large shoals, and even islands, in the center of the streams.

Where watermills exist there is no jurisdiction to compel the miller to maintain his works and regulate the weirs so as to give sufficient waterway in times of flood. Water-power is too valuable to be done away with, and the holding up of the water is a great advantage to the locality; but the owner should be placed under such restrictions that his weir and by-passers should not be of sufficient capacity, and he should not be allowed to interfere with the efficient discharge of the water during floods.

In like manner the weirs belonging to the navigation need remodelling, and the works to be placed under an efficient system of supervision along the whole river.

The effect of the floods of recent years has been most disastrous to the owners and occupiers of land from the losses they have incurred, and to the nation generally from the immense amount of produce destroyed. Thousands of acres of corn have been ruined by the summer floods, and land has been put out of cultivation by floods in the winter. The hay crops have been floated off the meadows and carried down the rivers, and a large area of rich pasture land has been so long inundated that the herbage has been rendered valueless. Additional taxes have also to be levied to pay for breaches in the river and drain banks caused by the floods, and for the maintenance of steam-power to pump the water off the flooded lands. It is not easy to calculate the loss which has been incurred during the last few years, but it certainly very far exceeds any sum required to place these rivers in a satisfactory condition.

REMEDY.

The works necessary for the prevention of floods in these rivers require to be carried out on a comprehensive scheme, commencing with the outfall and working upwards throughout the whole length of the channel.

The four rivers here specially referred to, discharging into the head of a bay or estuary abounding in shifting sands, are liable to have their mouths choked. The conflict between the ebb current and the flood invariably has a tendency to throw up a bar at the point where the confined channel debouches into the open. All works of improvement in the way of training and confining the channels ought therefore to be progressive and continuous, gradually pushing the confined channel forward to deep water.

In carrying out these training works the walls require to be at such a height and width as to prevent any retardation or choking of the tidal flow. The object to be sought is to give a free action to the tidal current as the principal agent in maintaining these channels in their most efficient condition, and to ensure that the last of the ebb shall be directed along a definite channel, so as to take every advantage of its scouring power. For this purpose the width of the channel should decrease from the sea grad-

ually, and the training walls, commencing at the lower end with a height equal to low water of neap tides, should, as they advance, reach to that of half-tide level.

Already the outfalls of the Nene and of the Ouse, which had been trained to deep water, are encumbered with sand. In the Nene the depth of water at the end of the trained channel has gradually decreased from 9 feet to 2 feet, the depth in the trained portion being 8 feet. Across the outfall of the Ouse there is a sand-bar, with only a depth of water 5 feet against 9 feet in the trained channel. In both cases the training requires to be carried seaward slowly, but continuously, or the advantages gained will disappear. The Welland discharges into a sand bed four miles distant from deep water; in fact, it may be said that when the water leaves the fascine work it no longer has any defined channel, but meanders over the sands, continually shifting its course. The Witham is in the same condition, but works are now being executed to carry the channel to deep water.

Notwithstanding the bars forming at the mouths of the Nene and the Ouse, the advantage of the improvements already effected in the outfalls of those channels is shown by a comparison of the level of low water in floods with that of the Witham. Taking each river at a point 8 miles from the estuary, the average level of low water of the same flood over a period of seven days was 16 feet 6 inches above low water of spring tides in the estuary in the Witham; in the Nene 7 feet 7 inches above, and in the Ouse 5 feet 6 inches above; showing a difference of 11 feet in the low-water level between the Ouse and the Witham.

The author has not been able to collect sufficient data to form any definite opinion as to the result of the works carried out in these rivers in raising or lowering the level of high water; but by a comparison of four years' tides at Lynn and Boston, it appears that mean high water is about 4 inches higher at Lynn than at Boston, which would show that the proper regulation of the channel has not a tendency to lower the high-water mark.

The value of tidal waters in maintaining the channels of these rivers in an efficient condition is of the utmost importance; and the deductions drawn from observations lead the author to an opposite conclusion to that laid down in the paper by Mr. W. R. Browne, M. Inst. C.E., on the relative value of tidal and upland waters in maintaining rivers, estuaries, and harbors. It is not contended that the enclosure of the marshes reclaimed by the training works has had any material influence on the outfall, the silting up of which is, as already explained, due to other causes, and would equally have taken place had these marshes remained open; but for the maintenance of the channel a free flow and ebb of the tidal water up and down the river is essential to prevent the sand carried up with the tide from being deposited. So long as the water is in motion only a small portion of the sand which is held in suspension settles; but where there is an obstruction to the tidal flow, and the water remains quiet, the heavy particles at once begin to sink and accumulate. In summer, when the flow of fresh water is small, this deposit remains. The quantity of water at spring tides in the embanked channels of these rivers is ordinarily six times as great as the upland water, and, being always in motion, must therefore have a greater effect in maintaining the channels of the rivers. The Ouse is in the best condition to allow a free run of tidal water; the Witham the worst. In the former river the tidal flow is 40 miles, and, even in the driest season, scarcely any silting up of the reaches of the channel occurs. In the latter the tidal flow is only 7 miles, the tide being stopped by a sluice; the deposits have been so great in dry seasons as to raise the bed of the river upwards of 11 feet at the upper end, and an average of 8 feet over the whole length of the trained portion of the channel, leaving upwards of 1,500,000 tons of silt and sand to be washed out by the winter floods, which have had to rise nearly high enough to submerge the country before they could flow over the deposit. In the Welland, which has a smaller drainage area, but a tidal flow of 20 miles, during the same season the depth of the deposit left at the head of the tides did not amount to more than 2 feet 6 inches.

Following the improvement of the out-

fall, the channel requires regulating throughout its whole length by widening and deepening in parts and confining the low-water level where too wide so as to give a general uniformity throughout. Too great a width impedes the free discharge almost as much as where the channel is too restricted. By the diminution in the velocity of the current owing to the greater capacity, deposits take place and shoals are formed through which the water continually alters its course as the ebb or the flood current is the stronger. In the marsh cut of the Ouse the banks have been gradually washed away, and the channel has become considerably wider than in the trained portion below; consequently shoals are forming, and the section of the channel has become very irregular, causing disturbance and increased friction and restricting the area of discharge.

Where the water is held up in the upper reaches, the weirs should be adapted to the largest flood discharge, as should all bridges and other structures across the waterway. While sufficient waterway should be secured for all floods, the low water channel should be so restricted as to maintain its scouring power in the fullest efficiency. It is in the adaptation of the channel to the normal flow, and also to the flood discharge, that the greatest difficulty occurs. The proportion between the one and the other, even in the flat district of the river basins here dealt with, may be taken as 10 to 1. Extreme floods occur only at uncertain and distant intervals. During the last thirty years there have been only twelve floods in this district which have done any serious amount of damage. Therefore if the channels be made sufficiently capacious to carry off these, they would be far too large for the ordinary discharge, and would become choked with shoals and weeds. The great expense and waste of land which would result from a channel made sufficiently capacious to carry off excessive floods, at once show that any such idea is impracticable.

In river improvement it must always be a matter of consideration whether the advantage to be gained by any particular scheme will be equal to the outlay, and whether it be not better to allow tracts of low-lying land, which are now occasionally flooded, to remain so, than to spend more than the value of their fee simple in protecting them. As pasture land they would always have a certain value, and where the owners have broken up such tracts into arable land, they have done so knowing the risk, and should abide by it.

A careful investigation into the rainfall in the Witham basin of the last fourteen years tends to the conclusion that the height of the floods is not entirely due to the actual amount of rain falling, as much depends on the condition of the land and other circumstances prevailing at the time. Taking the rainfall of Boston as typical of that of the Witham and Welland basins, a fall of 2½ inches in three days in July, 1867, only raised the water in the main drains 3 inches, whereas the same quantity, in July, 1872, made very heavy freshets in the river, and in July, 1880, caused a serious flood. Again, in 1868 although the rainfall for the autumn was heavy and continuous, and 6 inches above the average, yet the water in the Witham had not risen to flood height until the end of December. On the other hand, a fall of 1.66 inch of rain and snow in January, 1867, rapidly filled the rivers and flooded a considerable area of fen lands, although the rainfall for the previous period had not been excessive.

It has generally been the custom in designing fen drainage to allow at the rate of a continuous fall of 0.25 inch of rain during twenty-four hours. This calculation was adopted by Sir John Hawkshaw, Past-President Inst. C.E., in his report for the discharge of the whole basin of the Witham, and also for the large pumping engines at Lade Bank, for draining the East Fen. Sir John Coode, in his scheme for the improvement of the North level drainage in the Nene, provided for 0.25 inch, although he considered 0.187 inch would be all that would come daily to the outfall. During floods he ascertained that a quantity equal to 0.10 inch over the whole area of 79,855 acres was daily discharged.

During the last few years, the rainfall in the Witham district, if taken over seven days, would give a daily mean of 0.37 inch, or if over fourteen days, 0.25

inch, the maximum for the seven-day period being 0.63 inch, and for the fourteen days, 0.34 inch. Although at such times the ground is fully saturated, and in an exceptional condition, it is not possible that the whole of the rain which falls could be delivered at the outfall. The mean discharging capacity of the four rivers is equal to 0.094 inch every twenty-four hours, allowing a velocity of 3 feet per second (about 2 miles an hour).

To adapt the channel to the discharge of 0.25 inch in twenty-four hours would therefore require that they should be made nearly three times their present size, a course which, even if practicable, would render them far too large for all ordinary discharges. Provision for a continuous discharge of 0.25 inch of rain every twenty-four hours would require, with a velocity in the channel of 3 feet per second, a sectional area equal to 1 square foot for every 285 acres, whereas at the present time there is only an average of 1 square foot to every 816.6 acres.

The present discharging capacity of the Witham is equal to 0.105 inch of rain in twenty-four hours; of the Welland, 0.096 inch; of the Nene, 0.063 inch; and of the Ouse, 0.101 inch; and this is not sufficient to prevent flooding.

It becomes, then, necessary first to provide a channel for the ordinary discharge of the river, and also for occasional excessive floods. A modification of the system of wash lands, already referred to, points to the method of securing this end. The ordinary channel of a river should be of sufficient capacity to take the normal flow of the stream, the sides being made at as steep a batter as the natural inclination of the soil would allow, and at such a height as may be desirable for retaining the water for the supply of agricultural and domestic purposes or water-power. The water being then retained in as small a compass as possible, the weeds would be less likely to grow and shoals to accumulate. The sides beyond this should be laid at a slope sufficiently flat to allow of the growth of grass and the feeding of sheep and cattle in summer, and the protecting banks set sufficiently far back to allow room for the passage of the greatest floods likely to occur. Where banks already exist, they would require removing on one side at least, and where there are no banks the material dredged and cleaned out of the channel would in many cases be sufficient to form them. Bridges and other openings must, of course, be adapted to the flood discharge. By this means provision would be secured for both ordinary and flood-water, without loss of productive land, and the varying character of the discharge accommodated.

Where the channel passes through a town, as the Witham at Boston, the Welland at Spalding, and the Nene at Wisbech, the difficulty of altering the river is no doubt greatly enhanced; but it may be overcome in the manner proposed by Mr. Abernethy for Wisbech, by making an entirely new cut for the river, and dockizing that portion of the old river which passed through the town. By this means the discharge of the floods would be provided for, and by removing the ships from the channel where they are always an obstruction in floods, they would be enabled to lie and discharge afloat in the dockized channel of the old river at the existing granaries and warehouses.

It may no doubt be urged that the expense of thus altering and adapting a river to meet ordinary flood discharges would be very great, but if the cost was equitably spread over the whole watershed, the tax would not be greater than the advantage gained.

In the upper reaches of the river much flooding could be saved by dredging and cleaning out the present channels, and using the material in forming embankments, provision being made for the lateral drainage by soak dykes or drains parallel with the embankments, and discharging at a level sufficiently far down the river.

REGULATION AND STORAGE OF THE WATER.

The regulation of the water requires as much consideration as its discharge. The greater rapidity with which the rainfall is now avoided leaves less to percolate through the soil for the supply of wells, springs, and brooks. Flooding is thus frequently followed by drought. The level of the water in the soil is lowered below the depth at which it can rise

by capillary action to the roots of the plants, the soil becomes parched, and vegetation languishes for want of moisture, and great inconvenience is experienced from the failure of the water supply from wells and brooks.

In all river improvements the fact should be kept steadily in view, that the rainfall is only to be got rid of after making due provision for water supply, irrigation, water-power and navigation. These are none of them incompatible with good drainage. It is only necessary that proper provision should be made by sluices and weirs for the discharge of floods, and by side cuts or arterial drains where the water has to be held up so high that drainage cannot be obtained for the ordinary discharge.

The value of holding up the water as an aid in the cultivation of the soil is fully recognized throughout the whole of the Fens, as also in Holland. The water in the main and subsidiary drains is maintained in summer at a uniform level of from 2 to 3 feet below the surface, by a system of sluices with doors over which any surplus flows, but which are drawn immediately the supply exceeds the demand, and the water is thus regulated to a uniform level.

Water held up in a similar manner in the higher levels would not only feed the wells but afford power for the working of the machinery of the farms through which it traverses of a far more economical character than steam.

CONSERVANCY.

The administration of a river is hardly an engineering matter; but it is a subject which seriously affects the carrying out of any scheme of improvement. One difficulty encountered by an engineer is the restricted character of the portion of the river he has to deal with. He is called upon to devise a remedy against flooding or other evils in a particular section of a river, the remedy for which can only effectually be found by dealing with portions beyond the jurisdiction of those who have sought his aid. Attempts to bring the various bodies having control over the river into harmony, in order to carry out one comprehensive scheme, almost invariably end in failure from the diversity of interests. Every local scheme is violently opposed by all other interests; and it has been stated on reliable authority that the internecine feuds on the River Nene alone during the last fifty years have cost more than £100,000 in parliamentary and legal contests. The cost of obtaining the parliamentary powers necessary for the improvement of the Ouse have amounted during the past fifty years to upwards of £150,000; and for parliamentary proceedings alone for the Nene Valley Acts over £30,000.

An engineer is thus frequently compelled to design and execute partial works on a section of the river at great cost, where the same amount contributed to a general improvement would have effected tenfold advantage. Thus, on the Witham, within the last few years, a sum of nearly £50,000 has been expended on the middle section of the river in deepening the channel and raising the banks between Boston and Lincoln, without any provision for increasing the discharging power through Boston to the sea, or relieving the lands above Lincoln by enlarging the capacity of the weirs and sluices. This was done in spite of the protest of Sir John Hawkshaw that no effectual relief could be given without extending the works downwards to the outfall in the sea. The consequence of this action has been that the water is brought more rapidly to the lower reaches without being provided with any increased means of escape, and backs up the lateral drains, bringing greater pressure on their banks than they can bear. The floods have been greater in this district since this work was done than they ever were before.

It is only after repeated attempts, spread over the last eighty years, that the various trusts below Lincoln have at length united in a common scheme for the improvement of the outfall from Boston to the sea. Provision is also about to be made for the better discharge of the water from the river above Boston, but even now this will give little relief to the City of Lincoln and the lands above.

The same process took place on the Nene. A sum of £150,000 was spent in improvements on a section of the river between Wisbech and Peterborough; and the channel was lowered and

deepened without providing for the escape of the water to the outfall, the consequence being that the excavation rapidly filled up, and, in spite of this large expenditure and the consequent heavy taxation, no benefit ensued.

In the attempt made a few years ago by the corporation of Wisbech to carry out the scheme for cutting off the Horse Shoe bend through the town of Wisbech—a plan which had been recommended by every engineer who had reported on the matter for the last fifty years—they were defeated by the opposition of other interests in the river each fearing some damage to the particular section of the river or interest represented.

The number of private Acts of Parliament in force with relation to these four rivers, even only where they pass through the Fen land, is extraordinary. The number of jurisdictions which have control over the river or the banks has accumulated till at times it is almost impossible to define their powers and rights.

The whole history of the Fen land drainage shows the baneful result of divided administration, and teaches that no voluntary or private legislation is sufficient. The administration of the several districts protected by Fen Acts is most efficient so far as it goes, and some of the schemes in force may well form a model for any Conservancy Act that may be framed. To supersede existing organizations by new boards elected on a different plan would be most injudicious. What is wanted is a consolidation of all these smaller trusts, and the uniting them by representatives sent to one common Conservancy Board, which should have control over the main river and its banks from its source to the sea, leaving the management of the interior drainage to the trusts already in existence, or, where none exist, to others formed under the powers of the Land Drainage Act. Such a system would cause as little disturbance with existing arrangements as is practicable with an efficient system of conservancy of the main outfall.

MODERN ARTILLERY.

From "Engineering."

The present moment, when a large sum has been voted in the Budget for the partial re-armament of our Navy with guns of new type and of greater power, seems a fitting one for discussing those points of progress which have rendered such a re-armament not only desirable but necessary. The past four or five years have led to an increase in power of ordnance greatly exceeding anything ever before achieved in a similar period. The question of breech-loading *versus* muzzle-loading, certainly as far as naval guns are concerned, has been definitely decided in favor of the former system; and the causes of the settlement of this question, involving as it does the total renewal of our naval armament, are not far to seek, and are intimately associated with the increase in power of which we have spoken above.

In this country Sir William Armstrong, and in Germany Krupp of Essen, have taken the lead in progress. On the questions of difference between these great rival firms as to material and construction we will speak later. The general principles which have guided them in their remarkable and successful endeavors to increase the powers of modern ordnance may be briefly summed up as follows: From the results obtained by the Government Committee on Explosives and the researches of Abel and Noble on fired gunpowder, it became apparent that a high initial velocity of the projectile, together with its attendant advantages of flatness of trajectory, accuracy, power of penetration, and length of range could only be satisfactorily obtained by generating in the bore of the gun a large quantity of gas at low maximum tension or pressure. The production of a large quantity of gas can only be effected by using large charges of powder. A reduction of the maximum pressure may be secured by using either very slow burning powder, which be-

comes converted into gas at a much lower rate than is the case with the powder already in use; or by using the latter to reduce their destructive action by allowing the charge to expand in a chamber very much larger than is absolutely necessary to contain it. This latter method is technically known as air spacing. It is evident that a combination of both these devices is possible. The immediate result of the employment of either or both is to necessitate the use of very long guns, so as to keep the projectile in the bore under the influence of the propelling power of the gas for as long a time as possible, thus counteracting or more than counteracting the want of high initial pressure. The whole result may be described as follows: It has been found possible by the use of very slow burning powder, or of a quicker burning powder duly air spaced, and expanded in a very long bore to about double the power of ordnance weight for weight, and such a result does not seem to point to any finality in the path of artillery progress.

Now, departing from mere theory and passing on to the more practical application of the principles enunciated above, we find, broadly, that very similar results have been arrived at with guns manufactured on the following systems:

(*a*) Built-up guns, all steel. Types—Krupp, Vavasseur.

(*b*) Built-up arms of wrought iron with steel tubes, such as the Armstrong and Woolwich.

(*c*) Built-up guns with steel hoops and tubes, but depending for their main strength on steel wire of very high ultimate strength, wound on cold. Types—some French, American, and the Armstrong ribbon guns.

Before proceeding further we may say that we have already decided the question as to whether the gun should be a breechloader or a muzzleloader, laying it down as an axiom that our modern gun must have great length of bore. This of itself necessitates a breechloader. Ships cannot be built, or existing forts cannot be altered, in such a manner as to render it possible to work a muzzleloader of the length which is necessary to achieve the ballistic results now attained. And here we may clear the ground by observing that breechloaders cannot be double-loaded, as was claimed to be the case in the melancholy catastrophe of the bursting of the 38-ton muzzleloader in the fore turret of the Thunderer. A breechloader of the same size and weight as a muzzleloader entails much less labor to work than the latter; no sponging-out is required; the gun can be loaded at any position of training, and run out the gun's crew are much better protected against the fire of shrapnel, machine guns, and rifles; and finally, guns of greater weight can be manipulated by hand alone when loaded at the breech than at the muzzle. Practically it was found that in the case of the 38-ton R. M. L. guns the limit of size capable of being worked by hand had been reached, and complicated hydraulic or steam gear to assist manual labor became a necessity. In this country breechloaders of 43 tons in weight have been rapidly and easily worked by hand, and abroad Krupp's 70-ton has given equally satisfactory results. In fact, as weight increased, the muzzleloader became the more complex machine of the two.

All these reasons are independent of the great and paramount necessity for breechloaders arising from great length of gun.

Having decided then that our guns are to be breechloaders, we pass on to consider the question of their construction. It is a well-known fact that a solid homogeneous cylinder subjected to a heavy internal pressure, may be destroyed by the interior layers of the metal being strained above their ultimate tenacity, while the outer are hardly called on to do any work; this is more especially the case with a suddenly applied pressure such as that exerted by fired gunpowder. Hence arose the method, first practically applied in this country by Sir W. Armstrong, of putting on the outer portions of the gun in a state of tension, and as a consequence the adoption of the built-up systems of ordnance. This practice has become universal and has at present reached its furthest development in the wire or ribbon guns above mentioned.

When one tube is placed over another, the outer being in a state of tension, it is evident that the inner must be in a state of compression varying with

the amount of tension to which the outer is subjected and the relative thickness of the two tubes.

Thus, a gun theoretically perfect to withstand tangential or bursting pressure, should be built up of an indefinitely large number of very thin coils or tubes, each put on at such a tension that when a certain pressure is set up in the bore of the gun, the whole should be subjected to exactly the same strain, thus utilizing the strength of the material to the utmost.

Now, both theoretically and practically the above state of things is exceedingly difficult to arrive at. The earlier Armstrong guns had numerous very thin coils, and over and above the great cost of a structure built up in such a way, it was very difficult to regulate the exact amount of shrinkage to be given to each coil. Instances, indeed, did occur where the outer coils gave way without any damage occurring to the interior of the gun. The Woolwich system reduced the number of coils and thickened them, thus departing further from our ideal standard. Krupp first tried guns of steel cast in one solid mass; they naturally failed, and he eventually approached more and more to the methods adopted in this country, without, however, ever abandoning his material, viz., steel; Vavasseur alone, as far as we are aware, of English makers, following his example.

Modern experience tends to show the soundness, both in theory and practice, of the system of building up.

All ordnance now manufactured of any great power consists of a steel tube surrounded by either massive wrought-iron coils, as in the Woolwich guns; lighter and more numerous coils, as in the Armstrong; steel tubes or hoops, as in the Krupp; or steel wire and hoops, as in the latest Armstrong. Even at Woolwich, the stronghold of wrought iron, the superior merits of steel appear at last to be acknowledged, and it seems probable that, after a few years, the use of wrought iron will gradually have disappeared.

Of the different methods employed in the above systems of providing for the somewhat opposing demands for longitudinal and tangential strength, and also as to the qualities of the rival metals, steel and wrought iron, we propose to speak on another occasion.

PILE-DRIVING FORMULAS AND PRACTICE.

By RD. RANDOLPH, C.E.

Written for VAN NOSTRAND'S ENGINEERING MAGAZINE.

IT is to be hoped that the directions given by the Chief of Engineers to those officers in charge of pile-driving operations for public buildings will have the effect of establishing a more certain guide for such work than the fallacious and conflicting formulas to be found in the text-books, and to which attention has lately been called by the article in the July number of VAN NOSTRAND'S MAGAZINE.

Whatever coefficients may be determined from these experiments, the formulas otherwise must depend upon a true theoretical deduction; and it is important that those who are to determine them see very clearly the truth of the theory before applying the coefficients as factors; not as Col. Comstock advises, "to take some good formula such as Rankine's," selected from the general assortment, perhaps upon the scientific standing of the authority, or because it has heretofore been generally adopted.

From the fact that Col. Mason did not determine his formula until after completing the work at Fort Montgomery, it might be inferred that it was the result of his experience, but as the buildings have settled since it is evident that he did not have sufficient time to test practically its truth. But the Mason formula is the same as that of Weisbach, which is derived from purely theoretical considerations, and which falsely assumes that the resistance to the penetration of a pile is of the same character as that of the resistance of gravitation to a pro-

jectile in vacuo; and that the path described with an equal initial velocity against either, will be in proportion to the intensity of resistance to each particle of the masses.

By this assumption the inertia overcome by the pile while penetrating the mass of the earth, a force which increases with the velocity, is completely ignored. This can be proved by the formula itself; and for the purpose take this illustration. Suppose the pile to encounter at a distance from the surface another pile in the same vertical line, placed there by some pre-historic race, thus forming a continuous pile of greater length. The last blows of the hammer might cause an uniform penetration which would be used in the formula; but the application of it now would give the pile credit for supporting a greater weight than would be true, because one of the factors, the pile, has not been fully estimated. The unknown portion of the pile has absorbed the momentum of the hammer just as has the known portion of it. Although in practice such a case may never occur, yet there is always a mass of earth to be driven down, or laterally from the sides, and waves of concussion to be sent through the earth in all directions; an effect increasing with the velocity and generating the resistance of inertia, but unlike the hidden pile, it does not apply the momentum absorbed to the penetration.

In order to show the effect of ignoring the hidden pile let us apply a formula which ignores the inertia of both, as Rankine does. When the hammer falls from the height F, according to the well-established law of falling bodies, its velocity at the point of impact is

$$\sqrt{\frac{2F}{g}} \times g = \sqrt{2F} \times \sqrt{g}.$$

Now assume that the resistance to the further descent of the hammer is a force uniformly distributed, not through space, but through time, like that of gravitation, and which brings it to rest in a distance equal to the penetration denoted by p. If gravitation could be so magnified as to give that much resistance to the mass of the hammer, it would not ascend higher than p, although protected with a velocity of $\sqrt{2F} \times \sqrt{g}$. Likewise if let fall from the height of p, under the influence of the same force, its velocity at the end of p would be $\sqrt{2F} \times \sqrt{g}$. As the accummulated effect of one second's duration of actual gravitation is a velocity represented by g, let that of the resistance to the hammer be represented by x. Then, by the same law, $\sqrt{2p} \times \sqrt{x}$ is the velocity acquired or overcome, as the case may be, in the distance p, which we have seen is $\sqrt{2F} \times \sqrt{g}$. This gives the equation

$$\sqrt{2p} \times \sqrt{x} = \sqrt{2F} \times \sqrt{g}$$

which reduces to $x = \frac{F.g}{p}$. The intensity of the two forces being in proportion to the two velocities g. and $\frac{F.g}{p}$, acquired in the same time, their ratio is $\frac{F}{p}$; and the pile will resist $\frac{F}{p}$ times as much force as is offered to the hammer by actual gravitation, which is its weight, denoted by W. It will therefore sustain $\frac{W.F}{p}$ lbs. In the example at Proctorville, mentioned by Gen'l Weitzel, this would be $\frac{910 \times 5}{.03125} = 142400$ lbs., a result nearly reached by Rankine's formula.

But now supply the omission and include the pile, as is done in Weisbach's formula. The velocity of the hammer is the same as before $\sqrt{2F} \times \sqrt{g}$, but before the penetration begins, and at the moment of impact, its momentum is applied to both masses, and they move with a common velocity which is proportionably less; and is expressed by

$$\sqrt{2F} \times \sqrt{g} \times \frac{W}{W+w}$$

in which w represents the pile. The equation now becomes

$$\sqrt{2p} \times \sqrt{x} = \sqrt{2F} \times \sqrt{g} \times \frac{W}{W+w},$$

which reduces to $x = \frac{F \,.\, g \,.\, W^2}{p \,.\, (W+w)^2}$, this divided by g gives the ratio of $\frac{F \,.\, W^2}{p.(W+w)^2}$

and the resistance to the pile is equal to that many times the force of gravitation on hammer and pile, *i. e.*, their weight. Multiply this ratio by $W+w$ and we have $\frac{F.W^2}{p.W+w}$; which is the weight to be supported by the pile according to Weisbach. In Gen'l Weitzel's example this would be $\frac{5}{.03125}\times\frac{910^2}{910+1611}=52557.$

In both cases the masses were supposed to be resisted by an uniform force which in equal small divisions of time subtracted equal amounts of velocity, and that the paths described were the measures of the intensity of the resistance to each particle of the mass when referred to one due to the same initial velocity. Knowing the initial velocity of each and referring to the path that would be described if projected against gravitation with the same initial velocity, the relative intensity of the resistance in each case to that of gravitation was obtained; which being applied to the quantity of each mass determined the resistance in terms of gravitation. But as the paths are in proportion to the square of the initial velocities which produce them, the resistance to each particle is in the same proportion. Therefore the resistances are respectively in proportion to the square of the velocity multiplied by the mass; which accounts for the difference in the results. Diminished velocity does not compensate for a proportionate increase of mass.

This proves that the Weisbach formula, or any other which is deduced from the law of falling bodies, cannot be applied unless all the elements of inertia are represented and the velocity of penetration modified accordingly. We have seen the effect of omitting the pile, and can therefore appreciate the effect of omitting the hidden pile which the formula would not reach; and in the same manner we may comprehend the great variety of mass put in motion at every blow of the hammer and which no figures could fully express. And we can understand that most of this motion is wasted in producing other mechanical effects than contributing to the penetration of the pile.

So far has been considered only masses which share the momentum of the hammer before the commencement of the observed penetration; but such mass may be infinitely subdivided and uniformly distributed along the path of the penetrating body like the particles of a fluid. But in the same way the momentum may be divided into elements, each having its initial velocity to be affected in the manner observed in the case of the integer. Such a resistance may be resolved into elements of pure impact, which would show it to be in proportion to the square of the velocity. For the sake of illustration, suppose two locomotives to be running on parallel lines, one at double the velocity of the other, and they encounter a long drove of cattle standing equi-distant upon the track—the resistance to the first will be four times that to the second; because in the same space of time it collides with twice the number of objects and hurls them all with double the velocity. Instead of masses suppose the obstructions to be cords so light in proportion to their strength as to be devoid of inertia—the resistance to the first would be twice that to the second, because it would depend solely upon the number broken in a certain time. But suppose these latter to be equi-distant in time instead of space, the resistance would be equal to both locomotives, as they would encounter the same number in the same time. When the pile driver has to overcome a resistance like the last, a formula derived from the law of falling bodies can be applied. But when it is of a character of the two first, it must be so modified as to represent the relations of the elements of mass and velocity.

It is also to be noted that two quantities have been neglected in the Weisbach formula; one of them is small enough to be neglected, but the other has been recognized by Rankine. The first is the action of gravitation during the penetration which counterbalanced the resistance to that extent. This would require that the weight of the hammer should be added to the indicated load to be supported, as that does not remain with the pile. The second is the compression of the pile, which is a part of the penetration applicable to the hammer, while the observed penetration is applicable to both.

In the formula of Rankine this compression and the observed penetration are both applied to the hammer alone, as the pile is entirely ignored otherwise. The whole movement is supposed to be resisted by the friction of the earth along the sides of the pile; and all resistance to be independent of velocity. Thus differing from Wiesbach only in this, that the latter neglects elements of inertia that are not apparent, while Rankine neglects those that are apparent and great in quantity.

The example mentioned by Col. Tower will illustrate this error on an exaggerated scale. He supposes a heavy target suspended like a ballistic pendulum. If we press against it with the hand we will, under that slow movement, encounter only the resistance of friction at the point of suspension and a very slight effect of its gravity when pushed beyond the vertical. Now if a shot be fired through the target, does that shot have no other resistance than the friction at the point of suspension and gravitation along the very small arc through which the target moves, and which just before was over come by the pressure of the hand? Or was not the inertia developed by the high velocity of the projectile so great, that it was easier to tear away the solid metal than to overcome it to any considerable extent? So the compression of the pile is due to its own inertia developed by the velocity of the hammer, as well as the resistance of the earth behind it; the latter becoming less in comparison as the fall of the hammer or weight of the pile is increased.

In his "Applied Mechanics," Professor Rankine gives a formula for pile driving which results in a smaller quantity than the one given in his work on "Engineering," the difference being due to the modulus of elasticity being applied to one-half the length of the pile in the first and one-quarter of the length in the second. In order to see the elements considered, let us trace the process through which the formula is reached. The hammer being the only mass considered we have, as before, $\frac{W.F}{p}=R$, or $W.F=R.p$, denoting by R the resistance or the weight to be supported. The penetration is now increased by the compression of the pile, and which we will call c. This will give $W.F.=R.p+R.c$. As the modulus of elasticity, denoted by e, will compress one square inch of the sectional area of the pile, denoted by s, its whole length, denoted by l; R will compress it $\frac{R}{e}$ of its length, or $\frac{R.l}{e}$; but will compress the whole area only $\frac{R.l}{e.s}$; which is the value of c. But as the resistance is considered as distributed along the whole length, and not at one end, the compression will diminish from the full quantity at the top to zero at the bottom uniformly, and will amount to one-half of the full quantity for the whole length. In which case the value of c becomes $\frac{R.l}{2.e.s}$. By substituting this in the equation, it becomes

$$W.F=R.p+\frac{R^2.l}{2.e.s} \text{ or } \frac{R^2}{2.e.s}+\frac{R.p}{l}=\frac{W.F}{l},$$

or

$$R^2+R.\frac{2p.e.s}{l}=\frac{2W.F.e.s}{l}.$$

Complete the square of the first member of the equation by adding the square of one-half the coefficient of R in its second term to both members.

$$R^2+R.\frac{2.p.e.s}{l}+\frac{p^2.e^2.s^2}{l^2}=\frac{2W.F.e.s}{l}+\frac{p^2.e^2.s^2}{l^2}.$$

Then extract the square root of both members,

$$R+\frac{p.e.s}{l}=\sqrt{\frac{2.W.F.e.s}{l}+\frac{p^2.e^2.s^2}{l^2}},$$

or

$$R=\sqrt{\frac{2W.F.e.s}{l}+\frac{p^2.e^2.s^2}{l^2}}-\frac{p.e.s}{l},$$

which is Rankine's formula in "Applied Mechanics." But in his "Engineering," for some reason which he does not state, he considers the compression as applicable to only one-fourth of the length of the pile; making the value of c in the above $\frac{R.l}{4.e.s}$ which changes the final equation to

$$R=\sqrt{\frac{4W.F.e.s}{l}+\frac{4p^2e^2s^2}{l^2}}-\frac{2p.e.s}{l}.$$

Taking the same modulus of elasticity for both, 750 tons, or 1,680,000 lbs., and the other data in the example of General Weitzel, $l=30$, $s=138.25$. the indicated resistance by the first is 117,208 lbs., and by the second 128,530 lbs.

It will be seen that the only difference between these formulas and the one first suggested, $R=\frac{W.F.}{p}$, is the increasing the penetration by the extent of compression, and the effect of this is seen by comparing their results with that of the latter. which, with the same data, was 142,400 lbs.

The Weisbach formula depends upon the assumption that at the instant of contact the hammer and pile were endowed with a common velocity due to their combined masses; which could not be the case if the pile undergoes compression; for the hammer would move faster and the pile slower than this until the compression ended, the momentum of the two masses being the variable parts of a constant sum.

But the initial velocity of the penetration is less and combined with a less mass, since the momentum of the hammer is not all applied until the compression is exhausted. The addition of the whole mass during the penetration will compensate for its deficiency in the beginning, as far as momentum is concerned; but as time has been lost in its application, the deficiency of velocity in the beginning is not compensated for, as far as this effects penetration; for, according to the theory upon which both formulas are based, the resistance is distributed uniformly in time like that of gravitation — not uniformly in space. The penetration will therefore be less than that due to the assumed condition of inelasticity; and this will be assigned to greater resistance instead of less velocity. Any correction then, on account of compression of the pile, will diminish the result of the formula and take it still further from that of Rankine.

If all the elements of inertia could be as easily ascertained as the principal one, the inelastic pile, it would only be necessary to add the mass representing it to the pile in the formula of Weisbach. And perhaps experiments may determine the value of this quantity for different situations. But a very simple experiment will determine whether it can be correctly applied without this addition. Let the fall of the hammer be so adjusted that the initial velocity of hammer and pile in one case may be double that of another. If the penetration in the first is four times that of the second, it will prove that the law of falling bodies can be applied, otherwise not.

If it were true that the piles are supported only by the friction against their sides, each cluster would have to be considered as one pile, and the surface of the cluster would represent the resistance. Also the weight of the cluster would have to include the intervening material; for being cut off from *terra firma*, it would be supported by the piles alone. But this would imply that the base of the pile-work was a fluid which would receive the pressure, or a part of it, if the lateral friction was insufficient; and would yield, however slowly, until an equilibrium was established. It has been observed that sheets of lead that have remained for centuries upon the steep roofs of ancient buildings are very decidedly thicker at the lower edge than the upper; from which it is inferred that the flow of cold lead, like the flow of the glacier, is only a question of time. So that any test which might be made by placing a load upon a pile that has been driven, would fail to indicate, in the limited period at the disposal of the engineer, the extent to which it might yield after the lapse of years.

But however fluid the pile foundation may be, it can develop inertia under velocity which would completely falsify a formula which ignores that element—altogether absent in the case of a quiescent load.

A NEW VARIETY OF GLASS.—The *Wiener Gewerbe-Zeitung* states that a chemist of Vienna has invented a new kind of glass, which contains no silex, potash, soda, lime, nor borax. In appearance it is equal to the common crystal, but more brilliant; it is perfectly transparent, white and clear, and can be cut and polished. It is completely insoluble in water and is not attacked by fluoric acid, but it can be corroded by hydrochloric and nitric acid. When in a state of fusion it adheres to iron bronze and zinc.—*Gaceta Industrial.*

SUBSCALES, INCLUDING VERNIERS.

By H. H. LUDLOW, 2d Lieut. 3d Artillery, U.S.A.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

SUBSCALES IN GENERAL.

1. *Measurement* of distance is the determination of a required distance by comparing it with some known distance called the *unit of measure*. This comparison may be effected by successively applying the unit to the required distance, until the remainder is less than the unit. The remainder is then neglected altogether or considered as an additional unit, according as it is or is not less than half the unit. If a more nearly exact result is desired, a smaller unit of measure must be taken. This may be done, either by taking a smaller unit in place of the one first used and beginning the measurement anew; or, better, by treating the unit first taken as a collection of new units, simply measuring the remainder in terms of the *secondary* unit which should exactly divide the *primary*. In like manner remainders from the secondary unit may be measured in terms of a *tertiary* unit, &c. The smallest unit taken in any system of measurement is called the *ultimate* unit.

2. Standard distances for the measurement of other distances have been adopted and named, as an inch, a yard, a meter, &c. These are necessary to express a distance conveniently. They may or may not be the primary, secondary, &c., units of measure actually applied, and may be called for distinction *units of expression*.

3. For convenience, *scales* are frequently formed by the successive application of the unit along a line, so that any distance shorter than the scale may be measured at a single application. For distances longer than the scale, the whole scale may be applied as a primary unit of measure, the unit on the scale becoming secondary. The least space on the scale used is ordinarily taken as the ultimate unit of measure, but frequently it is taken as a primary unit, with a smaller ultimate unit. An auxiliary scale is then needed to measure[1] the remainders from the primary unit, and if applied directly[2] to the main scale and along[3] it, the auxiliary scale is called a *subscale*. Hence:

4. A *subscale* is an auxiliary scale of equal parts, directly applied along a main scale of equal parts: for measuring all distances along the latter, taking the least space on the main scale as a primary unit, with a smaller secondary unit. The least space on the main scale is called the *scale space*; that on the subscale, the *subscale space*. The secondary unit is called the *least count*.

5. When a division of the subscale is directly opposite a division of the scale, so that the two form one continuous line, the subscale division is said to *coincide*, and is called a *coincident division*.

6. In measurement two divisions are taken, one on each scale, and the distance between them made equal to that to be measured. The distance is then determined by a coincident division. In exact measurement a coincident division must exist, and the sum or difference of two distances, one on each scale, measured from it, will give the required distance. Every common aliquot part of the scale and subscale spaces exactly divides this sum or difference; and no distance thus exactly measurable can be less than their greatest common aliquot part.

7. All distances exactly measurable by this combination alone,[4] must, § 4, be capable of exact expression in terms of the secondary unit. But they may also be ex-

[1] The remainders may be estimated by conceiving the least space on the scale to be subdivided, but this is not in general reliable.

[2] As an illustration of indirect application, may be mentioned the scale on the head of a screw for measurement along the axis.

[3] The diagonal sliding scale is directly applied, but not along the main scale.

[4] If a tertiary unit were used, it would require, beside the scale and subscale, either an additional device for measurement or supplementary estimation.

pressed, § 6, in scale and subscale spaces. The secondary unit or least count must then exactly divide the subscale space as well as the scale space,[5] and cannot exceed their greatest common aliquot part. Nor can it be less than this part since it is exactly measurable by the combination. Hence, *the least count is equal to the greatest common aliquot part of the scale space and the subscale space.*

8. That each subscale division may in turn be coincident and opposite any scale division, the dividing lines on both scales must all intersect the line along which the scales meet. That the subscale shall always measure *along* the scale, the two scales must accurately fit each other, however placed, which condition limits the possible shapes of scale and subscale, in a plane, to the straight line aad arc of a circle.

9. *Relations of subscale elements.* — Those quantities which are always the same for the same scale and subscale are called subscale elements. In any scale and subscale, denote by l, a, b, the *least count, scale space,* and *subscale space* respectively; then, § 7,

$$l=\frac{a}{q}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

$$l=\frac{b}{q'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

$$\therefore \quad qb=q'a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

in which q and q' are whole numbers mutually prime, and $q>1$ since $l<a$, § 4.

Since the least count is the secondary unit, it is less, § 4, than the scale space. It must then, § 6, be the difference of two distances, one on each scale. Let r, r', denote the least numbers of subscale and scale spaces respectively that can differ differ by l. Then,

$$\pm l=r'a-rb. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Divide both numbers by l and reduce by (1) and (2), then

$$\pm 1=r'q-rq'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

r and r' are integers, also $r'q$ and rq' are mutually prime, their difference being unity; r' and q are each prime with respect to both q' and r, r cannot be o, since that would require $q=1$.

l, a, b, q, q', r, r' are subscale elements.

10. From equation (3) since q and q' are mutually prime:

1°. In every subscale q is the least number of subscale spaces that can exactly cover a number of scale spaces; and q' the least number of scale spaces that can be exactly covered by a number of subscale spaces.

2°. If any subscale division coincide, § 5, those subscale divisions separated from it by q, $2q$, $3q$, &c., subscale spaces, and those only will also coincide.

3°. If any subscale division fails to coincide with the nearest scale division by a given distance, the subscale divisions separated from it by q, $2q$, $3q$, &c., subscale spaces, will each fail to coincide with its nearest scale division by the same distance estimated in the same direction.

11. If r and r' are known, q' may be eliminated from (2) by (5) leaving in (1) and (2) four elements, any two of which will determine the others.

If r and r' are unknown, it will be shown § 43 that (5) suffices to determine them when the other elements are known. Ignoring r and r', (1) and (2) are independent equations, containing the five elements l, a, b, q, q', any three of which will determine the other two, provided the given quantities do not all enter the same equation. If l is given with a or b it must, § 7, be an aliquot part of each. In one case two quantities, a and b, suffice, owing to the fact that q and q' are mutually prime; for (3) may be written $\frac{q}{q}=\frac{a}{b}$, which in its simplest form gives both q and q'.

12. *Classification.*—Subscales are classified according to the relations between scale space and subscale space, as *simple, vernier,* and *complex* subscales, §§ 21, 26, 42. A subscale is *direct* when, of the least scale and subscale distances (4) differing by l, the greater is on the scale; *retrograde* when the greater is on the subscale.

13. Subscales are further classified according to their extent. A *complete* sub-

[5] This requires two commensurable scales. If incommensurable scales were used, no unit *secondary* to the scale space could exactly express all the distances exactly measured. The auxiliary scale would not be a subscale, § 4, and the combination would be very inconvenient.

scale is equal in length to the distance on the indefinite subscale, from any coincident, § 5, division to the next coincident one. A subscale of less extent is *incomplete;* of greater extent, *redundant.*

14. A *complete* subscale contains just q spaces, § 10, 1°. Redundant spaces are each separated by q, or $2q$, or $3q$, &c., spaces from some division among the first q; and those of each set of corresponding divisions, § 10, 2°, 3°, are like situated for coincidence. Measurements with subscale are based, § 6, on coincident divisions. Hence, redundant divisions do not in general increase the efficiency of a *complete* subscale. In any *complete* subscale we see (1) that the *least count is equal to the scale space divided by the entire number of subscale spaces.*

15. To decide whether a given subscale is *redundant, complete,* or *incomplete,* the definition may be directly applied, or the entire number of spaces may be compared with q if known. When the second coincidence exists, q and q' may be found by direct observation, § 10, 2°.

16. *Measurement* with scale and subscale consists of two parts: 1st. *Adjustment,* so that the required distance shall be equal to that along the scale from a division, usually the zero, to the zero division of the subscale. 2d. The *reading, i. e.,* finding the distance by inspecting[6] the adjusted scales.

17. *Adjustment.*—1st. The scale should be in such a position along the line of the required distance that its zero will be at one extremity of that distance. As the scale must keep this position throughout the measurement, it should, if practicable, be firmly fastened. 2d. The subscale should be in such a position along the scale that its zero will be at the other extremity of that distance. Accurate adjustment is usually effected by a rack and pinion, or by a clamp and screw device.

18. *Reading.*—The result of the act of reading is called the *final reading.* It expresses the distance from the scale zero to the subscale zero, and is composed of two parts, called the *scale reading* and *subscale reading,* and determined from the numbers on the scale and subscale respectively. The *scale reading* expresses the distance along the scale from its zero division to the division of reference, *i. e.,* the scale division to which the position of the subscale is referred. The *subscale reading* expresses the distance from the division of reference to the zero of the subscale.

For convenience the division of reference is so taken that the *final reading* shall always be equal to the arithmetical sum of the *scale reading* and *subscale reading.*

19. Let AD, Fig. 1, represent any scale with its zero at A, and let V be the position of the subscale zero after adjustment.

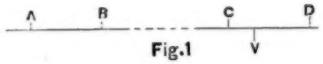


Fig. 1

The method of coincident divisions § 5 having been adopted, § 6, some division of the subscale must be coincident or be (provided the least count is to be the ultimate unit of measure) considered coincident with a scale division. The division which most nearly coincides is considered coincident.[7] If V is considered coincident let C be the corresponding scale division. C is then the division of reference, and we have subscale reading $=o$, scale reading $=$ AC $=$ final reading. If V is not considered coincident, let V lie between the consecutive scale divisions C and D. C lying on the side of the lesser numbers of the scale is then taken as the division of reference,[8] and we have scale reading $=$ AC, subscale reading $=$ CV, final reading $=$ AV $=$ AC $+$ CV. The subscale reading, CV, is then differently determined for the different classes of subscales.

20. If CV (Fig. 1) is determined directly from it, the subscale is said to be *forward arranged*; if indirectly from the relation CV $=$ CD $-$ VD, *backward* arranged.[9]

[6] A simple inspection is sufficient to determine the measurement, if the subscale is properly numbered.

[7] Compare note 4.

[8] D might have been taken as the division of reference. Then AV $=$ AD $-$ VD. This is inconvenient as the scale reading AD would have to be diminished by the subtraction and could not be at once written as a part of the final reading.

[9] The terms *forward* and *backward arranged* were first applied to verniers according to the direction in which it measures its own small motions, as compared with that of increasing scale measurements. See § 28.

SIMPLE SUBSCALES.

21. A *simple subscale* is one in which the *subscale space* exactly divides the *scale space.*

22. For simple subscales we have, § 7,

$$l=b \quad . \quad . \quad . \quad . \quad (6).$$

which in (4) requires $r=1$, $r'=o$, giving in (5)

$$q'=1 \quad . \quad . \quad . \quad . \quad (7).$$

a relation which also results from comparing (6) with (2). r and r' being known the other elements may be found as in § 11.

23. For *measurement*, the subscale zero should, according to § 16, be at the extremity of the required distance. But ordinarily a simple subscale is detached, and is used as may be most convenient. It is merely a scale of finer subdivision than the main scale, for measuring directly the distance from the extremity of the required distance to either of the two consecutive scale divisions between which that extremity lies. One of these consecutive scale divisions is the division of reference. Direct measurement to it corresponds to *forward* arrangement; direct measurement to the other scale division to *backward* arrangement.

24. Whether a simple subscale is *redundant complete* or *incomplete* may be decided as in § 15. Practically it is only necessary to compare its entire length with a scale space. If incomplete, it is too short to measure directly all fractional parts of the scale space. But it may be used whenever it is as long as half the main scale unit.

25. The simple subscale is inconvenient when the least count is very small, as the spaces may be too small for distinct vision.

VERNIER SUBSCALES.

26. A *vernier subscale* or vernier (so called from its inventor, Pierre Vernier of Brussels, A. D. 1631), is a subscale, in which the difference of scale and subscale spaces exactly divides the scale space.

27. The difference of vernier and scale spaces is their greatest common aliquot part, which fact requires, § 7,

$$\pm l=a-b \quad . \quad . \quad . \quad (8).$$

giving in (4) $r=1$, $r'=1$, and reducing (5) to

$$\pm 1=q-q' \quad . \quad . \quad . \quad (9).$$

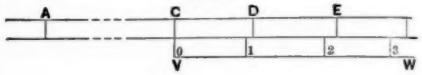
A vernier is *direct* or *retrograde*, § 12, according as $a>b$ or $a<b$.

Solving (9) with respect to q', we see, § 10, that:

Every complete vernier covers $q \mp 1$ scale spaces according as it is *direct* or *retrograde*. r and r' being known, the other elements may be found as in § 11.

28. *Reading.* Let AE (Fig. 2) be any scale VW any accompanying complete vernier. Resume the notation of § 9 and 27. Denote the vernier reading by x. Let the consecutive vernier divisions be numbered 0, 1, 2, 3, &c. to q beginning

Fig. 2



with V, which is supposed coincident with some scale division C. Since l is numerically equal to $a-b$, § 27, the vernier divisions numbered 1, 2, 3, &c. to q will fail to coincide with the corresponding scale divisions by l, $2l$, $3l$, &c., to $ql=a$. If the vernier be now moved in such a direction that the vernier division numbered 1 will at the outset approach its corresponding scale division, vernier divisions 1, 2, &c., will in succession coincide, § 5, and thereby measure the distances l, $2l$, &c., passed over by the vernier zero V. If at positions intermediate to those of exact coincidence the most nearly coincident vernier division is taken as coincident, the error cannot exceed $\frac{l}{2}$. This gives the required measurement in all cases to the nearest unit with l as the unit of measure. The distance thus directly measured is that from the scale division C to the vernier zero V in the position read, estimated in the direction of the supposed motion. If this motion is in the direction of increasing scale numbers, C is the division of reference, § 19, for the position read, the vernier is *forward arranged*, § 20, and we have

$$x=nl \quad . \quad . \quad . \quad . \quad (10)$$

If this motion is in the direction of decreasing scale numbers, the division of reference is the scale division next to C on the side of the lesser scale numbers, the vernier is *backward arranged*, and we have

$$x=a-nl=(q-n)l \quad . \quad . \quad . \quad (11).$$

In both (10) and (11) n denotes the number of the coincident vernier division.

29. *Forward* and *backward arrangement.*—If the vernier is direct, its spaces are smaller than the scale spaces, and the above-supposed motion is in the direction of increasing *vernier* numbers. A *direct* vernier, § 12, will then be *forward arranged* if its numbers increase in the same direction as the scale numbers; and *backward arranged* if they increase in a contrary direction. In like manner it may be shown that a *retrograde* vernier is *backward arranged* if its numbers increase in the direction of increasing scale numbers, *forward arranged* if they increase in the contrary direction.

30. The *vernier zero* is naturally taken at one of the extreme divisions, but in a complete vernier the zero may be at any intermediate division. It is only necessary that the divisions preceding the zero division shall be marked with the same numbers that they would have, if removed bodily and placed as redundant spaces at the end of the vernier. For any one of them can coincide, § 5, only when the corresponding redundant division coincides, § 10, 1°. This requires that the number on the last division shall be repeated on the initial division, after which the numbers increase in the same direction and by the same law as before.

31. *Measurement.* — The vernier is ordinarily forward arranged, for which arrangement the important steps in measurement are summarized in the following

RULE.

Adjustment.—The scale and vernier should be in such positions that the required distance shall be equal to that along the scale from its zero to the vernier zero, § 17.

Reading.—1st. If the vernier zero is considered coincident, § 5, read the corresponding scale division for the final reading, § 19.

2d. If the vernier zero is not considered coincident, § 19, read for the *scale reading* the division of the scale next to the vernier zero on the side of the lesser numbers of the scale. Then multiply the number of the vernier division considered coincident by the least count for the *vernier* reading; add the *vernier* reading to the *scale* reading for the *final reading.*

32. If the vernier is *backward arranged*, the vernier reading as found in the above rule must be replaced (11) by the remainder after subtracting it from the scale space.

33. Many verniers are marked with the numerical values of l, $2l$, &c., to ql on the 1st, 2d, &c., to qth divisions, thereby avoiding the multiplication in the application of the rule. Frequently also intermediate numbers are omitted, and divisions at regular intervals only are numbered.

34. There is difficulty in finding the most nearly coincident vernier division, when l is less than the width of the lines on the instrument. Thus if the nth vernier division coincides exactly, the $(n+1)$th appears also coincident, and so on in both directions until the difference becomes perceptible. The nth division is then the middle one of those apparently coincident, and their number is odd.

In reading such a vernier, take as coincident the middle one of the coincident divisions; if their number is even, either of the two middle ones may be taken with an approximate error[10] of $\frac{l}{2}$. With such a vernier a lens is frequently used to aid the eye, and a few redundant spaces are generally added at each end, so as not to diminish the number of consecutive coincident divisions, when the reading is near the end. The extremities of the vernier proper are then plainly marked.

[10] Whenever two consecutive vernier divisions are equally near to coincidence, the lesser reading may be taken, and $\frac{l}{2}$ added, thus rendering the result more nearly exact. Judgment might be further used to estimate a fractional part of l, but it is in general unreliable.

35. If the vernier is redundant or complete, the nth division considered coincident always exists since $n<q$, § 28. If the vernier is incomplete, the nth vernier division may be beyond its limits. Such a vernier is inconvenient for use. It is possible to use such a vernier, provided it contains at least $\frac{q}{2}$ spaces, but it must be *forward arranged* when the vernier reading is less, and *backward arranged* when greater than $\frac{a}{2}$.

36. *Classification.* — To determine whether a given subscale is or is not a vernier,[11] we have (9) which is more convenient than the direct application of the definition, § 26, when q and q' have been determined. If the subscale is known to be complete, § 13 and 27, it must, if a vernier, cover just one more or one less scale space than its own number of spaces, which fact is decisive and can be observed directly.

The condition for a direct vernier, § 27, is $a>b$ or $q>q'$; for a retrograde vernier $a<b$ or $q<q'$. Either form of the condition may be used according to the elements already determined. In practice the first form is the more convenient, and whether $a>$ or $<b$ may be directly observed. If l is very small, it may be necessary to look along the scale and vernier from the coincident division, until the aggregate difference is perceptible; if the greater aggregate distance is on the scale, $na>nb$ or $a>b$; if the lesser distance is on the scale $a<b$.

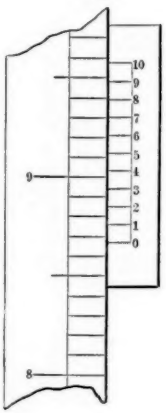
37. *Single, double, double folded.*—A *single vernier* is a complete vernier bearing on its divisions but one set of numbers (see § 38, 39).

Some main scales have the zero at an intermediate division, the numbers increasing in contrary directions from it. If a single vernier is forward arranged on one part, it will, § 29, be backward arranged on the other part. If one part is short and used only in detecting instrumental errors, the vernier is forward arranged for the longer part, and the shorter part is called the *scale of excess.* If both parts are to be used in measurements, backward arrangement is generally avoided by using two single verniers on opposite sides of a common zero, one for each part. The two verniers united form a *double vernier* (see § 40).

The same result of double reading may be attained with but one vernier, by giving on the same lines of division two sets of numbers increasing in opposite directions. An intermediate division, usually the middle one, is taken as the common zero of both sets of numbers which are arranged as explained in § 30. Such a vernier is called a *double folded vernier* (see § 41). It is more compact than the double vernier.

38. *Illustrations.*—One of the simplest of single verniers is represented in Fig. 3. The scale space is $\frac{1}{10}$ ft., and 10 vernier spaces cover exactly 9 scale spaces. The numbers on the scale correspond to tenths of a foot, and the part represented is supposed to lie between 4 and 5 ft. It is forward arranged, direct, and reads 4.867 ft. This is like the vernier on the "New York" leveling rod.

Fig. 3



39. The vernier of the ordinary cistern barometer is represented in Fig. 4. The scale space is $\frac{1}{20}$ inch, and 25 vernier spaces exactly cover 24 scale spaces, giving a direct vernier whose least count is $\frac{1}{500}$ in. $5l = \frac{5}{500}$ in. $= \frac{1}{100}$ in., and every fifth vernier division, § 33, is num-

[11] If $a-b=\frac{a}{2}$. $b=\frac{a}{2}$ and the subscale is at the same time a simple subscale and a vernier. It may he used either way.

bered. The vernier is forward arranged, and the third vernier division after the one numbered 1 is coincident. It reads $29.60 + .01 + 3 \times .002 = 29.616$.

The most common errors are to omit the first adjustment, § 17; and in reading to neglect one of the least spaces on the main scale, when the scale division read is not an even tenth of an inch.

least count is 1′. There are two sets of numbers, each increasing from 0 to 15 at one end, and then from 15 at the other end to the middle division. The division numbered 7 and 23 coincides. The reading is 1° 7′, 7 being the set of numbers giving forward arrangement, § 29. Such a vernier is in use on the vernier compass by W. and L. E. Gurley.

Fig.4

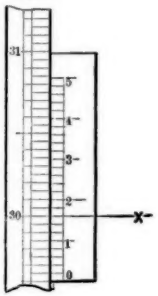
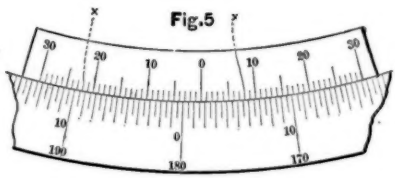


Fig.5



40. One of the simplest of *double verniers* is found on the Surveyor's Transit, by W. and L. E. Gurley. The scale space is $\frac{1}{2}°$ and each half of the vernier covers exactly 29 scale spaces. The least count is $\frac{1}{30}$ of $\frac{1}{2}°$ or 1′. It is a direct vernier, and as represented in Fig. 5, divisions 7 and 23, are coincident. The reading with the outer scale numbers is $177\frac{1}{2}° + 23'$ $= 177°\ 53'$; with the inner scale numbers

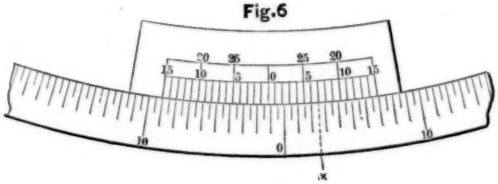
COMPLEX SUBSCALES.

42. A *complex subscale* is one which is neither *simple* nor *vernier*.

43. Equations (1), (2), (3), (4), (5), are applicable to *complex* subscales, provided $q > 1$, § 9, $q' > 1$, § 22, and either $r > 1$ or $r' > 1$, § 9, § 22, § 27.

If r and r' are known the elements may be determined as in § 11. For a

Fig.6



2° 7′. Care must be taken to read the half of the vernier which is forward arranged, § 29.

The corresponding retrograde vernier would cover 2×31 instead of 2×29 scale spaces.

41. A *double folded* vernier corresponding in use to that in Fig. 5, is represented in Fig. 6. It is, however, retrograde. The entire vernier of 30 spaces covers 31 scale spaces of $\frac{1}{2}°$ each. The

given scale and subscale q and q' may be found by direct observation, § 15.

If r and r' are both unknown, the other elements may be readily found, § 11, § 15. Equations (4) and (5) express the same relation and furnish a means of determining r and r' which are of use in numbering the subscale, § 46. (5) is the simpler form to use, and must, from its deduction, be capable of solution for each subscale. This does not further

limit the values of q and q',[12] and in each case one and but one set of integral value for r and r' not exceeding[13] $\frac{q}{2}$ and $\frac{q'}{2}$ respectively, can always be found, when q and q' are known. These

[12] To show that (5) is capable of solution with integral values of both r and r', no matter what mutually prime integers q and q' may be.

If $q=1$ or $q'=1$, the above statement is self-evident.

If $q>q'>1$, divide q by q' and continue the division until a remainder $d_1<q'$ is found. In like manner divide q' by d_1 with a remainder $d_2<q'$, d_1 by d_2, &c., until $d_n=1$, which must result (G.C.D.), since q and q' are mutually prime. Denote by $c\,c_1c_2$, &c., the successive quotients, then

$$\left.\begin{array}{l} d_1=q-q'c \ldots\ldots\ldots\ldots\ldots\ldots (1). \\ d_2=q'-d_1c_1 \ldots\ldots\ldots\ldots\ldots\ldots (2). \\ d_3=d_1-d_2c_2 \ldots\ldots\ldots\ldots\ldots\ldots (3). \\ \quad \&c. \\ d_{n-1}=d_{n-3}-d_{n-2}c_{n-2} \ldots\ldots (n-1). \\ 1=d_{n-2}-d_{n-1}c_{n-1} \ldots\ldots\ldots\ldots (n). \end{array}\right\} (a).$$

in which all the letters represent positive integers.

In (n) group (a), replace c_{n-1} by k_1

$$1=d_{n-2}k_1-d_{n-1}$$

then replace d_{n-1} by its value from $(n-1)$ group (a), giving

$$1=-k_1\,d_{n-3}+(1+k_1\,c_{n-2})\,d_{n-2}.$$

Let $k_2=1+k_1\,c_{n-2}$, and we have

$$-1=k_1d_{n-3}-k_2\,d_{n-2}.$$

In like manner combine this equation with $(n-2)$ group (a), denoting the new coefficient of d_{n-3} by k_3, &c., throughout group (a). The results may be written

$$\left.\begin{array}{l} 1=d_{n-2}-k_1d_{n-1} \ldots\ldots\ldots\ldots\ldots (1) \\ -1=k_1d_{n-3}-k_2d_{n-2} \ldots\ldots\ldots\ldots (2) \\ 1=k_2d_{n-4}-k_3d_{n-3} \ldots\ldots\ldots\ldots (3) \\ \quad \&c. \\ (-1)^{n-2}=k_{n-2}q'-k_{n-1}\,d_1 \ldots (n-1) \\ (-1)^{n-1}=k_{n-1}q-k_nq' \ldots\ldots\ldots (n) \end{array}\right\} (b).$$

in which

$$k_1=c_{n-1}.\quad k_2=1+k_1c_{n-2}.\quad k_3=k_1+c_{n-3}.\quad \&c.$$

$$k_{n-1}=k_{n-3}+k_{n-2}\,c_1.\quad k_n=k_{n-2}+k_{n-1}\,c.$$

We may then determine k_1, k_2, &c., to k_n, which are all positive integers. Comparing (n) group (b) with

$$\pm 1=r'q-rq'. \quad . \quad . \quad . \quad . \quad (5).$$

we see that $r=k_n$, $r'=k_{n-1}$, will satisfy it.

If $q'>q>1$ an equation analogous to (n) group (b) may in like manner be found, and a set of positive integral values for r and r' in (5) determined.

[13] Let s, s' represent any known set of integral values for r and r' respectively in (5), then

$$\pm 1=s'q-sq'. \quad . \quad . \quad . \quad . \quad . \quad (c).$$

Adding nqq' to, and subtracting it from the second number of (c) we have

$$\pm 1=(s'+nq')q-(s+nq)q'. \quad . \quad . \quad (d).$$
$$\pm 1=(s'-nq')q-(s-nq)q'. \quad . \quad . \quad (e).$$
$$\mp 1=(nq'-s')q-(nq-s)q'. \quad . \quad . \quad (f).$$

Comparing (d) (e) (f) with (5) we see that

From any set s,s', of positive integral values of r and r' in (5) other such sets may be formed by adding nq and nq' to, or subtracting nq and nq' from s and s' respectively, n being any integer. Positive results belong (d) (e) to the same $(\pm)$ form as s and s', negative results with their signs changed (f) to the opposite form.

This is the law of formation of all possible integral roots of (5); for let t, t' be any other set of such roots, then

$$\pm 1=s'q-sq'. \quad . \quad . \quad . \quad . \quad . \quad (c).$$
$$\pm 1=t'q-tq'. \quad . \quad . \quad . \quad . \quad . \quad (g).$$

If both sets belong to the same form the signs of the first members are alike, and

$$o=(s'-t')q-(s-t)q'.$$

or
$$\frac{s-t}{s'-t'}=\frac{q}{q'}.$$

in which since s, s', t t', q q' are integers, and $\frac{q}{q'}$ is irreducible.

$$s-t=nq. \quad . \quad . \quad . \quad . \quad . \quad . \quad (h).$$
$$s'-t'=nq'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (k).$$

n being some integer.

If the two sets of roots belong to opposite forms

$$o=(s'+t')q-(s+t)q'.$$

or
$$\frac{s+t}{s'+t'}=\frac{q}{q'}.$$

and
$$s+t=nq. \quad . \quad . \quad . \quad . \quad . \quad (l).$$
$$s'+t'=nq'. \quad . \quad . \quad . \quad . \quad . \quad (m).$$

in which n is some integer.

In both cases t and t' may be formed from s and s' respectively by the above law.

We are now ready to show that:

One and but one set of integral values for r and r' in (5) not exceeding $\frac{q}{2}$ and $\frac{q'}{2}$ can always be found.

From the above law, one set and but one in *each form* of (5) can always be found not exceeding q and q' respectively. Let c, c' denote the least set for the first form d, d', that for the second form, then

$$1=c'q-cq'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (n).$$
$$-1=d'q-dq'. \quad . \quad . \quad . \quad . \quad (p).$$

also $c+d<2q$, and $c'+d'<2q'$, which requires (l), (m)

$$c+d=q. \quad . \quad . \quad . \quad . \quad . \quad . \quad (r).$$
$$c'+d'=q'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (s)$$

are the least possible integral values of r and r' in (5), and are the required values § 10. To find them, substitute for r' [or r] in (5), 1, 2, 3, &c., in succession, deduce each corresponding value of r [or r'] until an integral result is obtained. The integral values of r and r' so obtained are the required values, if they do not exceed $\frac{q}{2}$ and $\frac{q'}{2}$ respectively; if either exceeds, subtract them from q and q' respectively, the remainders will then be the values required.

If r and q' [or r' and q] are given, $r'q$ may be found from (5), and r', q, will be integral factors of the product. Each set of such factors, satisfying the conditions $r=$ or $<\frac{q}{2}$, $r'=$ or $<\frac{q'}{2}$, q and q' mutually prime, will give a subscale.

If r and q [or r' and q'] are given, the values of r' and q' may be found in the same manner as those of r and r' when q and q' are given. The above conditions must, in any case, be satisfied.

44. Reading. Let l, a, b, q, q', r, r', be the subscale elements, as in § 9, x the subscale reading for any complete subscale. Conceive an auxiliary scale and subscale formed by erasing on the scale and subscale all lines of division except on the scale, the division of reference and those divisions separated from it by r', $2r'$, &c., scale spaces; and except on the subscale, its o, r, $2r$, &c., divisions. Denote by a', b', the new scale and subscale spaces respectively. $a'=r'a$. $b'=rb$; whence (5).

$$\pm l=a'-b'.$$

If $c<d$, (v) gives $c<\frac{q}{2}$, which in (n) gives $c'<\frac{q'}{2}+\frac{1}{q}$; but c' is an integer, and $q>1$ (for complex subscales), so that $c'=$or$<\frac{q'}{2}$. c and c' do not exceed $\frac{q}{2}$ and $\frac{q'}{2}$ respectively, while $d>\frac{q}{2}$.

If $c=d$, (v) gives $c=\frac{q}{2}$, and from (n), $c'=\frac{q'}{2}+\frac{1}{q}>\frac{q'}{2}$, which in (s) gives $d'>\frac{q'}{2}$, with $d=\frac{q}{2}$.

If $c>d$, (v) gives $d<\frac{q}{2}$ and (p), $d'<\frac{q'}{2}-\frac{1}{q}$.

In each case one set of such values, and but one, can be found.

This shows the new subscale to be a vernier, § 27, whose least count is l. It is direct or retrograde with the given subscale. The subscale reading and vernier reading in any position measure the same distance, since the division of reference and subscale zero are in common.

Hence, § 28,

$$x=nl \quad . \quad . \quad . \quad . \quad . \quad (10).$$

or

$$x=a-nl=(q-n)l. \quad . \quad (11).$$

according as the vernier is forward or backward arranged, n denoting the order of the coincident vernier division. Since $ql=a$, q vernier spaces are sufficient. The nth vernier division is the rnth subscale division. If $rn<q$, the coincident division is on the complete subscale; if $rn>q$, the subscale divisions $rn-q$, $rn-2q$, &c., are also coincident, § 10, and some division of the complete subscale also coincides. Let this division be numbered n. It will then be only necessary to multiply n by l for the subscale reading in equation (10). This, § 20, corresponds to forward arrangement of the subscale. The distance is expressed, § 28, to the nearest unit with l as the unit of measure.

45. The conditions for forward and backward arrangement of the subscale, are the same as for the auxiliary vernier. A change in the direction of the numbering, § 29, reverses the arrangement.

46. The law of numbering imposed, § 44, on complex subscales requires that the subscale division coincident in the same position as the nth division of the auxiliary vernier, shall be numbered n, whatever value n may have from o to q. Every given vernier division has its corresponding subscale division separated from it by sq subscale spaces (s being an integer). If n is not o or q, there can be but one corresponding division on the complete subscale. Apply the complete subscale, whose length is qb, successively r times to the vernier whose length is $q\times rb$. At each application the subscale is moved q spaces. On the $(s+1)$th application the corrresponding subscale division will be superimposed on the given vernier division. All the numbers may be located in this way. Each application locates several numbers on the subscale.

Let $\frac{q}{r}=k_1+\frac{q_1}{r}$. $\frac{2q}{r}=k_2+\frac{q_2}{r}$. &c., to

$$\frac{rq}{r}=k_r+o=q.$$

1st. Numbers 0, 1, 2, &c., to k_1 are located on the o, r, $2r$, &c., to $k_1 r$ subscale divisions.

2d. (k_1+1), (k_1+2), &c., to $k_2=k_1+(k_2-k_1)$ on the $(r-q_1)$, $(2r-q_1)$, &c., to $[(k_2-k_1)r-q_1]$ divisions.
&c., &c., &c.

rth, $k_{r-1}+1$, $k_{r-1}+2$, &c., to $k_r=q$ on the $(r-q_{r-1})$, $2r-q_{r-1}$, &c., to q divisions.

47. The *position* of the *subscale zero* may be intermediate, as may be shown by replacing the word vernier by subscale in § 30. The non-consecutive numbering [14] renders such an arrangement more confining than on a vernier.

48. A rule for *measurement* with scale and complex subscale forward arranged, may be had from that of § 31 by replacing the word "vernier" by "subscale." The same change also renders § 32 and § 33 applicable to the use of complex subscales.

49. If l is less than the width of the dividing lines, several subscale divisions will be coincident at the same. They correspond to consecutive divisions of the auxiliary vernier § 44, but are separated by r subscale spaces or r' scale spaces. They are in general non-consecutive subscale divisions, consecutively numbered, and the middle one must be taken as coincident, § 34.

The comparison to determine which divisions are coincident is in general more difficult than on a vernier; for, being non-consecutive divisions, they are not so readily grouped by the eye. If $r=1$, $r'>1$, the coincident divisions are consecutive *on the subscale*, and this difficulty disappears. But the complete subscale covers $(r'q\mp1)$ scale spaces (5) while the equivalent vernier would cover only $(q\mp1)$ spaces (9). If $r>1$, $r'=1$, the coincident divisions are consecutive *on the scale*, and the difficulty also disappears. The complete subscale then covers $\left(\frac{q\mp1}{r}\right)$ spaces and is more compact than the equivalent vernier.

50. If the subscale is redundant or complete, the division numbered n will exist for all values of n (o to q), and every distance less than a can be directly measured by it. If the subscale is incomplete, the division numbered n may be beyond the limits of the subscale. Such a subscale cannot measure directly every distance less than a and is inconvenient for use. It may, however, be used, provided it contains $\frac{q}{2}$ or more spaces; for the numbers are interchanged on the two halves when the arrangement is reversed, § 45. It must be capable of use arranged either way.

51. *Classification.*—In all subscales, the scale and subscale spaces must, note 5, § 7, be commensurable.

To determine whether or not a given subscale is *complex*, apply the tests for simple subscales, §§ 21, 22, and vernier, § 36; if it is neither of these, it must be complex. When r and r' have been determined, the condition that one of them shall be greater than unity is more convenient. Whether the subscale is redundant, complete, or incomplete, may be decided as in § 15. The condition for *direct* subscales is $rb<r'a$, or $rq'<r'q$: for retrograde subscales $rb>r'a$, or $rq'>r'q$, § 12. Either form may be used.

Subscales may, like verniers, be further classified, § 37, as *single* and *double*. A *double folded* subscale would, however, be too complicated for convenient use, unless $r=1$.

52. *Illustration.*—Required a complete complex subscale to go with a main scale divided to $\frac{1}{4}$ inch, which will enable one to measure to $\frac{1}{100}$ inch. There are but two given quastities $a=\frac{1}{4}$, $l=\frac{1}{100}$, and the problem, §§ 11, 43, is indeterminate.

From (1) $q=\frac{a}{l}=25.$

Repeating (5).

$$\pm1=r'q-rq' \quad . \quad . \quad . \quad . \quad . \quad (5).$$

we see that if different values are given in succession to r', values of rq' will result, each of which, if composite, may be factored, giving sets of values of r and q', for each value of r'. b may be found (2) from q' and l. Each such set of values

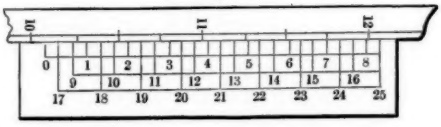
[14] The subscale might be numbered consecutively, and its reading found when any division coincides. But the operation of finding the reading is too complicated for convenient use.

will give a different required subscale, if $r=<\frac{q}{2}$ and $r'=<\frac{q'}{2}$, § 43.

In the most compact form, § 49, $r1'=1$, which in this case gives $rq'=25\mp1$. Taking the upper sign, § 42, for a *direct* subscale, we may write $r=3$, $q'=8$. The corresponding subscale is represented in Fig. 7. The subscale division 7 coincides, and the final reading is 10.07 inches.

The number 24 affords several different sets of factors, each of which could be used, giving a required compact subscale; but care should be taken not to make the subscale spaces too small for distinct vision. This compact subscale may be reduced to a vernier by simply dividing each scale space into r equal parts. The least count is also divided by r.

Fig. 7.



If the value of rq' in (5) had been prime we would have had either $r=1$ or $q=1$, reducing the compact subscale to the vernier, § 27, or to the simple subscale, § 22. The compact subscale is then impossible; thus, if $q=30$, and $r'=1$, $rq'=29$ or 31 (compare §§ 40, 41).

53. *Relative advantages of simple, vernier,* and *complex* subscales. It is essential to ease of reading, with any scale and subscale, that the coincident subscale division may be readily found; that the scale and subscale spaces shall be large enough to be distinctly seen; that the divisions of both scales shall be plainly numbered; and that the entire subscale shall not be too long to be critically viewed at a single glance of the eye. For convenience of record, it is further desirable that the scale space shall be a unit of expression, § 2, or some aliquot part of one.

Facility of finding the coincident divisions requires that any division in its vicinity, for at least one of the two scales, shall the more nearly coincide the nearer it is to the coincident division. This excludes from further consideration those complex subscales in which both $r>$ and $r'>1$, § 49.

With a given least count, the best of the convenient forms is determined by the cost of making the instrument.

1st. If the least count is large enough to be distinctly seen, it may be taken as the subscale space of a simple subscale. By taking the subscale as long as practical convenience will allow, the number of divisions on the main scale may be reduced to a minimum, thereby giving a less cost of manufacture than with any other convenient form of subscale.

2d. If the least count is to be as small as possible, consistent with convenience, the scale space should be taken of a size very near the limit of distant vision. With a vernier, the scale space and vernier space are nearly equal (8), and can be practically seen with equal distinctness. If the complete vernier is then made as long as convenience will allow, it will give a smaller least count than any other convenient form of subscale. For the subscale space, if different from the vernier space considered, must be appreciably different unless both $r>1$ and $r'>1$ a case already rejected; if appreciably less, it is inconveniently small; if appreciably greater, either the entire subscale is inconveniently large or the least count must be increased.

In all instruments for the accurate measurement of angles, the cost increases more rapidly with the radius of the measuring arc, than with the number of divisions on it, within the limit of distinct vision. The size is further limited for portable instruments by convenience of transportation. In all such instruments it is always desirable to have a minimum least count along the arc, and no other form of subscale can surpass the vernier.

3d. If the least count is too small to be taken as a subscale space, and not as small as practical convenience will allow, the vernier must be compared with those complex subscales in which $r=1$ or $r'=1$. Of these the shortest for any given scale

and least count is the compact subscale in which $r>1$ $r'=1$. If such a subscale can be found with its least space just large enough to be distinctly seen, and its length just within the limits of practical convenience, it will surpass all other forms of subscale. The equivalent vernier, r times as long, is beyond the limits of convenient length, while the equivalent subscale in which $r=1$, $r'>1$ is even longer than the vernier. Thus in (Fig. 7) the compact subscale is 2 inches long while the equivalent vernier is 6 inches in length. To constrct a vernier of convenient size would require with the same scale a greater least count; and with the same least count a greater number of scale divisions per inch and a less absolute number of subscale divisions, which if the scale were long would increase the cost.

The advantage of the compact subscale is limited to straight scales.

TRUSSES WITH SUPERFLUOUS MEMBERS.

By WM. CAIN, C.E.

Written for VAN NOSTRAND'S ENGINEERING MAGAZINE.

M. MAURICE LEVY in "La Statique Graphique," note 2 (Paris, 1874), has published a notable theorem concerning trusses with superfluous members, or those containing a greater number of pieces than statics alone can define the stresses in requiring a resort therefore to the theory of elasticity.

His conclusions are especially interesting as bearing upon the economy of such systems, and the writer therefore hopes that a *résumé* of his method may prove useful to American engineers.

The aim has been to give the essential features of Levy's demonstration, in all their generality, though with certain modifications, in as simple and elementary a manner as possible, without following in all cases the method of the author, besides illustrating with simple examples, worked out in sufficient detail to enable the reader to clearly appreciate the methods involved.

The investigation of trestle piers will likewise be entered into and proper stress diagrams given for usual forms, without superfluous bars, when acted on by the wind and the weight of truss and train, and certain objectionable features of trusses and piers will also receive attention.

§ 1.

The figures of trusses may be classified into *deformable*, or those whose angles can vary indefinitely, the lengths of the sides remaining the same, and *indeformable*, or those whose angles are determined when the length of the sides are known.

The latter class may be divided into two: those which cease to be indeformable when we suppress one of the sides, called *strictly indeformable figures*, and those containing more lines than are strictly necessary to define the figure when the lengths of the sides are given in order, called *figures with superfluous lines*.

A figure strictly indeformable contains just enough sides, so that if the lengths of these sides are given in order it may be constructed.

Let us call m the number of the sides and n the number of joints or apices of the figure.

Then in order to construct it, draw any line AB equal in length to one of the sides (Figs. 1 and 2).

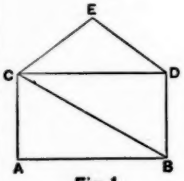


Fig. 1

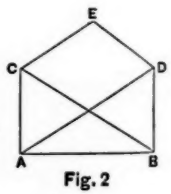


Fig. 2

Then describe two arcs of circles with A and B as centers with radii equal to AC and BC respectively. Their intersection will fix the position of joint C.

Similarly *each joint or apex* D or E is

defined by the intersection of *two sides and two only*, the sides being taken in the order assumed.

Therefore to each of the $(n-2)$ joints other than A and B correspond two sides so that the total number of sides of the figure, leaving out AB, is

$$2\,(n-2)=2n-4.$$

Hence the total number of sides including AB is,

$$m=2n-4+1=2n-3.$$

If the figure contains k more lines than the $(2n-3)$ corresponding to a strictly indeformable figure, these k lines are superfluous ("surabóndantes") to define the figure; in fact their lengths depend entirely upon the form and lengths of sides of the first figure, so that there must exist a geometrical relation between these lengths.

Again it is evident that if we suppress some of the sides of the first figure, that the resulting figure is deformable and can be constructed in an infinite number of ways.

From what precedes we see that we can always recognize the three classes by the following simple relations between the number of sides (m) and the number of apices (n).

For *deformable figures* . .	$m<2n-3$
" *strictly indeformable figures*	$m=2n-3$
" *figures with superfluous lines*	$m>2n-3$

It must be carefully noted that these relations suffice to distinguish the three classes *only* when the parts into which a figure may be divided, as well as the whole figure, belongs to the same class; otherwise, it can easily happen that part of the figure may have too few lines to strictly define it and another part too many lines, so that if the relation $m=2n-3$, for the whole figure was fulfilled, it would seem to indicate that it was strictly indeformable, whereas it is made up of figures belonging to the two other classes. The relations above then must not only be proved true for the whole figure, but for any and every part into which the figure may be supposed to be divided.

See Bow's "Economics of Construction" for a large variety of figures belonging to the various classes mentioned.

§ 2.

It is a well-known fact that when statics alone determines the stress in any bar, it does so irrespective of the *section* of that bar and consequently of its change of length after stress.

Therefore in a frame in which the stresses of the bars have been determined by statics alone, we can vary the sections of the bars, and consequently their alteration in length under stress, indefinitely, provided rupture does not occur, without the stresses being altered in the least.

Consequently *each bar must be free to alter its length irrespective of the changes in lengths of the other bars in order that statics alone can define the stresses*, for these stresses, once found by statics, remain unalterable, whilst the changes in length under stress vary with the sections, which we can choose at pleasure.

This *necessary condition requires* that the figure considered may be strictly geometrically determined when we know the lengths of the sides in order, and that it does not contain any superfluous lines, whose alterations of length are dependent entirely upon the alterations in length of the other sides. A system of lines, such as we are considering, can be constructed after strain exactly in the manner shown for (Figs. 1 and 2), taking the intersection of the new sides in order to fix the positions of the various apices.

We can proceed in another way to show that this requirement is not only *necessary* but *sufficient*. Thus, consider a frame subjected to external forces at the joints in equilibrium. At each of the n joints we can write two equations, representing that the sum of the horizontal as well as of the vertical components of the forces there, including the stresses of the bars, are zero. This gives $2n$ equations, but as there are three necessary equations between the equilibrated external forces, indicating that the sum of their horizontal components is zero, the sum of their vertical components is zero and their resultant moment about any point is zero, these $2n$ equations reduce to $(2n-3)$ independent equations.

Now this is just the number of sides $(2n-3)$ for a "strictly indeformable" figure, so that there are just as many equations as stresses to determine, and such figures, therefore, can be statically

determined. If there are more lines than the $(2n-3)$, there are too few equations by the excess, and the figure cannot be statically determined.

For a deformable figure there are more equations than necessary and the equilibrium is impossible unless the figure is given such a form that the external forces hold it in equilibrium.

We can state, therefore, the following theorem:

THEOREM I.—*In order that statics can furnish the stresses in a system of bars, it is necessary and it suffices that the geometric figure formed by the axis of these bars may be such that we can construct it, by giving in order the lengths of all the sides.*

If the figure contains k *superfluous lines, statics will furnish* k *equations too few to define the stresses in the bars; and inversely, if statics gives* k *equations too few to define the stresses we are certain that the figure contains* k *superfluous lines.*

Levy establishes this theorem very simply by the consideration of the principle of mutual velocities, which principle enables statics alone to determine the stresses whenever the figure is such that we can give to any one bar a small virtual elongation without changing the lengths of the other bars. We have seen above, that this condition is fulfilled when the figure has no superfluous lines.

§ 3.

This principle applies not only to *free systems*, but likewise to trusses, some of whose apices are subjected to certain conditions, provided these conditions affect only the position of the truss in space without influencing its form, so that each bar remains free to change length independent of the other bar. In this case, for figures in a plane, statics furnishes the reactions at the supports, so that the figure can be considered as free and subjected to the original forces to which are added the reactions of the supports. If this condition is not fulfilled, as for a truss continuous over several supports or for trusses fixed in direction, as well as position at certain points, as the braced arch fixed at the ends, etc., statics will furnish too few equations to determine the reactions at the supports by the number of extra conditions over those specified above.

In fact, statics furnishes three equations to determine the reaction at the supports, viz., (1) that the sum of the vertical components of the exterior forces, including the reactions equals zero; (2) that the sum of their horizontal components equals zero, and (3) that the sum of the moments of these forces about any point in the plane of the forces equals zero.

So that if a truss is fixed at one point, which involves two conditions (namely the two co-ordinates of the point), and free to slide at another point along some surface, curved or plane, which entails one condition or ordinate, in all three conditions, then statics will furnish as many equations as there are conditions, so that the reactions may be found and the figure be regarded as free.

But if we suppose the second point fixed, as well as the first, this will entail four conditions; so that statics will furnish one equation too little to determine the reactions. If three points are fixed, statics will furnish three equations too little, and so on.

In the case of the continuous girder, one joint is fixed at one support and the truss rests upon rollers at the other supports, so that statics furnishes too few equations by the number of intermediate supports.

In any case we can readily recognize whether the truss has more sides than is strictly necessary to build up the figure, knowing the length of the sides, considering the conditions to which it is subjected. If it has, then, by the theorem just enunciated, statics alone cannot ascertain the stresses.

In all cases, therefore, whether of trusses with superfluous bars or of trusses having more conditions to fulfil than are strictly necessary to define the form, knowing the length of the sides, statics furnishes too few equations by the number of the superfluous bars or of the extra conditions.

We must then resort to the theory of elasticity to furnish the extra equations needed, which may always be found, for whether we consider a truss with k superfluous bars or one subjected to such conditions that its form can be fully defined by leaving out k bars, there are always,

necessarily, k geometrical relations between the lengths of the bars, and therefore k equations between their elastic changes of length, which k equations added to the $m-k$ equations furnished by statics give as many equations as the number of the bars, so that the stresses in the bars can be fully determined.

We have seen that for figures with no superfluous lines or conditions, that the strains are independent of the sections of the bars and of the consequent elongations or compressions of the bars. If more bars are added than strictly necessary to define the figure, considering the conditions, the *changes of length* resulting from stress in all the bars depends entirely upon the geometrical relations of the sides, and the stresses in the bars depend upon these alterations in length, having assumed their sections and moduli of elasticity.

This is a marked difference in the two classes of trusses and must be carefully borne in mind in what follows.

Definition.—Where a truss is subjected to such *conditions*, that its form may be fully defined by leaving out k bars, these k bars are superfluous, in fact, to define the form, and we shall extend the definition of § 1 and class such trusses as belonging to systems with k superfluous lines.

§ 4.

General method for finding the stresses in the bars of a truss when statics leaves the problem intermediate.

Consider a truss with k superfluous bars, or one subjected to so many conditions that the figure is strictly geometrically defined when k bars are omitted, so that it really has k superfluous bars, as just defined.

First write the $(m-k)$ relations furnished by statics. Now there exists k geometrical relations between the lengths of the bars, giving therefore the lengths of k of the bars from the knowledge of the lengths of $(m-k)$ bars. Call

$$a_1, a_2, a_3, \ldots,$$

the lengths of the m bars in the natural unstrained state.

Under the influence of the forces applied at the joints of the truss, these bars take the elongations

$$\alpha_1, \alpha_2, \alpha_3, \ldots,$$

If any of the bars are compressed the corresponding α will be regarded as minus.

Since we have k geometrical relations between the lengths, let

$$F(a_1, a_2, a_3, \ldots,)=0 \quad \ldots \quad (1).$$

be one of them.

When the bars take the increments of length, this relation becomes

$$F(a_1+\alpha_1, a_2+\alpha_2, a_3+\alpha_3, \ldots,)=0 \quad (2).$$

If we call f the stress in a bar, $w=$ section, $e=$ coefficient of elasticity, $a=$ original length of bar and α its increase in length from the stress f, then we have, from the fundamental equation of the theory of elasticity,

$$\alpha=\frac{af}{ew} \quad \ldots \quad (3).$$

On subtracting (1) from (2), neglecting differences of a higher order than the first, which may be permitted in view of the limit of approximation permitted in the theory of elasticity, and substituting the value of α from (3) for each bar, we have *one* of the k equations sought.

Similarly the whole of these equations may be found.

These k equations thus obtained, joined to the $(m-k)$ equations furnished by statics, gives m equations, which are sufficient to determine the stresses in the m bars.

It may be remarked that it would be erroneous to *assume* the stresses of k of the bars, so that with the aid of the $(m-k)$ equations of statics, the stresses of the others could be determined, for from (3) the stress in any bar, $f=ew\frac{\alpha}{a}$ depends on the modulus of elasticity, the section and elastic elongation for unit of length, so that without considering the deformation of the whole truss or the relative elongations of the bars, the stresses cannot be correctly found, since each elongation depends upon certain other elongations.

We may express the method to be followed (see preceding page), in another manner.

Thus write Eq. (1)

$$F(a_1, a_2, a_3, \ldots) = F = o,$$

then by the theory of homogeneous functions

$$\frac{dF}{da_1}a_1 + \frac{dF}{da_2}a_2 + \ldots = o$$

On substituting the values of a_1, a_2, . . . from (3), we have one of the k relations sought

$$\frac{dF}{da_1}a_1\frac{f_1}{e_1w_1} + \frac{dF}{da_2}a_2\frac{f_2}{e_2w_2} + \ldots = o. \quad (4).$$

Similarly we find the remaining relations.

§ 5.

As an example illustrating the method to be followed, consider in Fig. 3, a system consisting of four bars, proceeding

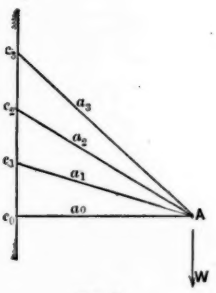


Fig. 3

from four fixed points c_0, c_1, c_2, c_3, in a vertical wall, to a common point A, where a weight W is applied.

The distance $c_0c_1 = c_1c_2 = c_2c_3 = b$; the lengths of the bars respectively are, a_0, a_1, a_2, a_3; their angles with the horizontal $\beta_0 = o$, β_1, β_2, β_3; and their stresses f_0, f_1, f_2, and f_3 respectively.

As there is only one joint A, statics can furnish but two equations,

$$f_1\sin\beta_1 + f_2\sin\beta_2 + f_3\sin\beta_3 - W = o \quad . \quad (5).$$

$$f_0 + f_1\cos\beta_1 + f_2\cos\beta_2 + f_3\cos\beta_3 = o \quad . \quad (6).$$

These two equations by themselves can only determine the stresses when the number of the bars is two.

It is seen that two of the bars alone fix the position of the point A, so that there exists a necessary relation between the lengths of the remaining bars and of the first two.

Now between the lengths a_1, a_2, a_3, we have the relation,

$$a_1^2 + a_3^2 = 2a_2^2 + 2b^2,$$

and calling the elongations under strain of the bars whose lengths are a_0, a_1, a_2, a_3 respectively, α_0, α_1, α_2, α_3 respectively, we have after the elastic deformation,

$$(a_1 + \alpha_1)^2 + (a_3 + \alpha_3)^2 = 2(a_2 + \alpha_2)^2 + 2b^2;$$

subtracting the former equation from the latter, and neglecting the squares of the elongations, we have,

$$a_1\alpha_1 + a_3\alpha_3 = 2a_2\alpha_2.$$

Or introducing the values furnished by eq. (3), we obtain, as one of the required relations,

$$a_1^2\frac{f_1}{e_1w_1} + a_3^2\frac{f_3}{e_3w_3} = 2a_2^2\frac{f_2}{e_2w_2} \quad . \quad . \quad . \quad (7).$$

The same result can be obtained by use of eq. (4).

In a similar manner, we should find,

$$a_0^2\frac{f_0}{e_0w_0} + a_2^2\frac{f_2}{e_2w_2} = 2a_2^2\frac{f_1}{e_1w_1} \quad . \quad . \quad . \quad (8).$$

These last two equations added to the first two furnished by statics, give four equations to determine the stresses in the four bars.

As before observed, these stresses depend upon the sections assumed or given. Thus with a given set of bars, whose sections are w_0, w_1, w_2, w_3, and moduli of elasticity e_0, e_1, e_2, e_3, respectively, we readily find from the 4 equations, the stresses f_0, f_1, f_2, f_3, by successive elimination and substitution. These stresses are thus found as numerical quanities, where tension is plus, and compression minus, from whence the stress per unit, $\frac{f}{w}$ for each bar can be determined.

By varying the sections we thereby vary the value for the stresses, which can thus be altered indefinitely, and in fact changed from tension to compression or the reverse in some cases. We thereby see the great influence of each section on all the stresses for systems not statically determined.

If the object is not simply to know the stresses in a given frame of the form considered, but to *design* the frame, so that the unit stress $\frac{f}{w}$ shall be a certain amount (which may be different for each bar), we must substitute the values of $\frac{1}{e}\cdot\frac{f}{w}$ for each bar in eqs. (7) and (8).

The result will show the geometrical relation that must exist between the lengths of the bars, in order that the hypothesis may be realized.

In case the relation does not show or lead to an absurdity, when the proper *signs* have been given to the stresses, *always agreeable to the laws of statics*, the system may be constituted with the kind of stress and the unit stress for each bar as assumed.

This part of the subject will be more fully treated in discussing systems of equal resistance.

As a second example take the figure formed by a rectangle and its two diagonals, not connected where they cross, and capable of taking both tension and compression.

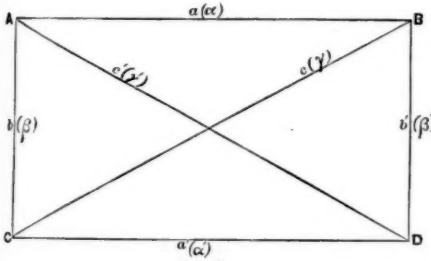


Fig. 4

Here we have $n=4$ joints and $m=6$ bars, so that $m>2n-3$ and the figure has one superfluous line.

Suppose forces applied at the four joints A,B,C,D, to hold the figure in equilibrium.

At each apex, statics furnishes 2 equations between the external forces and the stresses of the bars, in all 8 equations, but as the four forces satisfy 3 equations of equilibrium, these 8 reduce to 5 independent equations, or one equation too little to determine the stresses in the 6 bars.

To find the 6th equation, we resort to the geometrical relation between the lengths of the sides, in conjunction with eq. (3).

Thus call $a=a'$ the original or unstrained length of $\overline{AB}$ and $\overline{CD}$, α and α' their elastic elongations; $b=b'$, the primitive length of $\overline{AC}$ and $\overline{BD}$, β and β' their elongations; $c=c'=\sqrt{a^2+b^2}$, the length of the diagonals and γ and γ', their elongations, as marked on the figure.

We have

$$c^2=a^2+b^2$$

After deformation, this relation can be expressed in four different ways, according to the sides considered. Subtract the first equation from each of the four in turn, neglecting the squares of the elongations, add the results and divide by 4; we obtain,

$$c(\gamma+\gamma')=a(\alpha+\alpha')+b(\beta+\beta') \quad . \quad . \quad . \quad (9).$$

By aid of (3), this eq. is transformed to another, which in connection with the 5 eqs. given by statics, suffices to determine the stresses in the 6 bars.

If the sections of a frame of this kind are given, we find the stresses (plus or minus) from the previous equations from whence the unit strain $\frac{f}{w}$ for each bar is ascertained.

Where a figure of this kind constitutes one of the panels of a Pratt truss, the bars $\overline{CD}$ and $\overline{AD}$, say are in tension, and $\overline{AB}$, $\overline{AC}$ and $\overline{BD}$ compression. Let us ascertain whether $\overline{CB}$ is stretched or compressed.

Eq. (9), now takes the form

$$c(\gamma+\gamma')=a(\alpha'+\alpha)-b(\beta+\beta') \quad . \quad . \quad . \quad (10).$$

Let us suppose a common modulus of elasticity for all the bars and denote the stresses in the bars AB, CD, AC, and BD by f_1, f_2, f_3, f_4, respectively, and the corresponding sections by w_1, w_2, w_3, w_4; then by the use of eq. (3), (10), becomes

$$c(\gamma+\gamma')=\frac{a^2}{e}\left(\frac{f_2}{w_2}-\frac{f_1}{w_1}\right)-\frac{b^2}{e}\left(\frac{f_3}{w_3}+\frac{f_4}{w_4}\right) \quad . \quad . \quad , \quad (11).$$

A quantity essentially negative; for as there is generally but a small difference in the stresses of the chords AB and CD,

the quantity (a difference) inside the first [ ] is genrally small compared with the quantity (a sum) inside the second [ ]; consequently $(\gamma+\gamma')$ must be negative, but as γ' was assumed positive, it follows that γ must be negative and numerically greater than γ'; so that CB must be shortened when AD is lengthened as assumed. Therefore, if the bar CB is of such a small section, that it can receive no appreciable compression, it must be considered as out of action altogether, so that the system becomes statically determined.

In the Howe truss the diagonals can only receive compression, as their ends are simply butted against angle blocks, and we can prove for this truss in a similar manner that when one diagonal acts the other does not act, so that this system can likewise be statically determined.

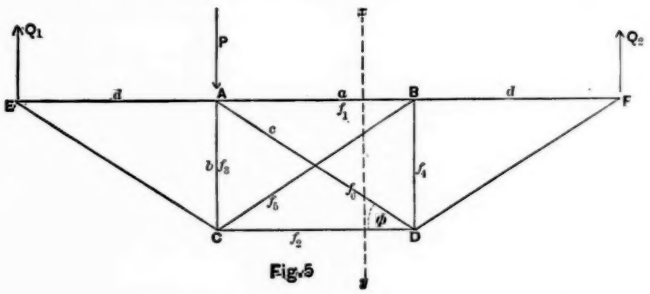


Fig. 5

It is well to call attention to these important distinctions, for they do not seem to have occurred to Levy, who classes all trusses having crossed diagonals with figures "*a lignes surabondantes.*"

Thus in the next figure, representing the ordinary queen post truss, we shall suppose the diagonals capable of taking either compression or tension at pleasure (which is not the case in American practice as just stated), so that the figure has one superfluous line, and statics will furnish one equation too little to determine the stresses.

With one weight P, applied at the joint A, the reactions Q_1 and Q_2 at E and F are found by the law of the lever and the stresses in the four extreme bars EA, EC, BF, and FD, follow from the ordinary laws of statics. We have thus the stresses in these bars together with the weight P taking the place of the external forces previously supposed to act at the four joints of the rectangle.

To find the stresses in the 6 remaining bars, it is simpler, as Levy remarks, in place of writing the $2n$ equations for the 4 joints, as above, to use the method of moments, in conjunction with that of sections, so that we write at once the 5 equations furnished by statics.

Call the stressses in bars AB, CD, AC, BD, CB, and AD, $f_1, f_2, f_3, f_4, f_5, f_6$, respectively, and their corresponding sections, $w_1, w_2, w_3, w_4, w_5, w_6$. We shall regard the modulus of elasticity the same for all the bars, and write the equations as if all the bars were in tension, since the plus or minus sign found finally for any stress, from the resulting equations, will show whether the bar is in tension or compression.

Suppose a section xy to cut the four bars shown and that the right part of the figure is in equilibrium under the action of the stresses in the four cut bars and of the reaction Q_2.

Taking moments about the point B we have

$$(f_2+f_6\cos\Phi)b-Q_2d=o \quad . \quad . \quad . \quad (12).$$

Calling φ the angle ADC and d the distance BF.

Similarly, taking moments about D,

$$(f_1+f_6\cos\Phi)b+Q_2d=o \quad . \quad . \quad . \quad (13).$$

Next balance the vertical components of the stresses at the section xy with the reaction Q_2,

$$(f_6-f_5)\sin\varphi=Q_2 \quad . \quad . \quad . \quad (14).$$

Now express that the vertical components of the stresses meeting at the point B are in equilibrium,

$$f_6 \sin\varphi + f_4 = o \quad . \quad . \quad . \quad (15).$$

The analogous projection for the point A gives,

$$f_6 \sin\varphi + f_5 = -P \quad . \quad . \quad . \quad (17).$$

These are the five equations, involving the stresses of the six bars, furnished by statics.

The sixth equation needed is obtained, as was Eq. (11), only regarding all the alterations in length as positive or elongations.

$$c^2\left(\frac{f_5}{w_5} + \frac{f_6}{w_6}\right) = a^2\left(\frac{f_2}{w_2} + \frac{f_1}{w_1}\right) + b^2\left(\frac{f_3}{w} + \frac{f_4}{w}\right) \quad . \quad . \quad . \quad (17).$$

By elimination between these six equations, having given the sections w_1, w_2, . . ., we find the stresses (plus for tension, minus for compression) in the six bars, and subsequently the unit stress $\frac{f}{w}$ for each of them.

This truss is usually designed, with such small sections for the diagonals, that the stresses in the other members of the rectangle are such as statics alone would give provided one diagonal was left out, *i.e.*, the top chord and posts in compression, the bottom member in tension. If we suppose one of the diagonals to take tension, the other, as we have seen, will take compression, so that Eq. (9), can be written for the most usual case,

$$c(\gamma' - \gamma = a)(a' - a) - b(\beta + \beta') \quad . \quad . \quad . \quad (18).$$

We may anticipate the next section, for this case, by asserting that this truss, deformed in the manner assumed, can never be made one of *equal resistance;* for in such forms, we shall find further on, that the changes in length per unit of length must be the same for each bar.

This amounts in this figure to making $\gamma = \gamma'$, $a = a'$, and $\beta = \beta'$, which reduces Eq. (18) to

$$o = o - 2\beta b,$$

which is absurd.

In fact it may be shown (see Levy's note) that on any supposition, agreeable to the laws of statics, of the signs of the stresses in the six bars considered, the system cannot be made one of equal resistance.

Where a number of rectangles with two diagonals each, like Fig. 6, are placed side by side, the diagonals being capable of taking tension and compression, we have a form of truss with as many superfluous lines as rectangles.

The preceding methods can be applied to each rectangle in turn, so that the stresses in all the bars can be found. It is evident how much we gain in simplicity by constructing the truss, so that the diagonals can only take one kind of strain, and since the former systems cannot be made of equal resistance, for any given loading, we should expect no economy in their use, as indeed will be demonstrated later for all systems with superfluous bars in the exceptional case where they *can be* constituted systems of equal resistance.

§ 6.

SYSTEMS OF EQUAL RESISTANCE.

In designing certain frameworks, we generally require that all the bars in tension shall be subjected to a certain unit stress and that all bars in compression shall sustain a certain other unit stress.

If the modulus of elasticity is not the same for the bars compressed as for those in tension, we may require that the stress per unit $\frac{f}{w}$ multiplied by the reciprocal of the modulus $\frac{1}{e}$, may be certain amounts for the bars in tension and in compression; so that *for all bars in tension,*

$$\frac{f}{ew} = \textit{elongation per unit of lengh} = c' \quad . \quad . \quad . \quad . \quad (19).$$

and *for all bars compressed,*

$$\frac{-f}{ew} = \textit{shortening per unit of length} = c'' \quad . \quad . \quad . \quad . \quad (20).$$

c' and c'' being certain *numerical constants.*

We regard here, as formerly, compressions as minus tensions.

The unit stress, $\frac{f}{w} = ce$, varies now with the modulus of elasticity.

Such systems will be called systems of equal resistance.

Now if we wish to ascertain the conditions that a system of bars should satisfy in order that they may be constituted a system of equal resistance, for the loading considered, we must substitute in Eq. (4), the values (19) for bars in tension, and the values (20) for bars compressed.

Let us designate by the subscript i, that the corresponding bars are in tension and by the subscript j, that the bars considered are in compression; then on substituting the values (19) and (20) in the k equations (4), the k equations that result can be put under the following form:

$$c'\Sigma\frac{d\mathrm{F}}{da_i}a_i - c''\Sigma\frac{d\mathrm{F}}{da_j}a_j = o \quad . \quad . \quad . \quad . \quad (21),$$

the first Σ referring to all the bars extended, the second to all the bars compressed.

Equation (21) represents one of the k equations of conditions.

Now we do not know in advance which bars are compressed and which extended; in fact the laws of statics will admit of a great many combinations, and each of these combinations will give a particular system of Eq. (21); but in order that the system of equal resistance may be possible, it is necessary that one at least of these combinations may be satisfied and that the signs of the stresses resulting must be as assumed in Eq. (21).

In fact we cannot, even when the equations of statics are satisfied, arbitrarily assume the signs of the stresses of but (m−k) *of the bars, for the* k *equations* (21) *determine themselves the signs of the other stresses.*

The most natural combination, and the one which the constructions would generally justify is that in which the signs of the stresses of the $(m-k)$ bars are such as statics would give if the k superfluous bars were removed.

If we multiply equations (19) and (20) by the lengths, a' and a'' of the corresponding bars, we have for the bars in tension, the total elongation,

$$a' = c'a' = \text{a constant} \quad . \quad . \quad . \quad . \quad (22).$$

and for the bars in compression, the total shortening,

$$a'' = c''a'' = \text{a constant} \quad . \quad . \quad . \quad . \quad (23).$$

It is therefore a distinctive characteristic of systems of *equal resistance*, that the total alterations of length remains the same for each bar, however the forces or sections may be varied.

If we vary the section of one of the bars and its consequent stress $f = cew$, we therefore change the stresses and consequently the sections of all the other bars; but if the signs of the stresses remain the same, the elongations per unit of length and also the total elongations of the bars are exactly the same as before, as follows from the preceding equations, and every supposition as to the sections of the bars embraces this hypothesis.

Therefore we may vary the sections indefinitely and consequently the stresses, provided the signs of the stresses resulting are such as assumed, agreeable to the laws of statics, and the system will still remain one of equal resistance.

We can thus announce the following theorem:

THEOREM II.—*In order that a system with* k *superfluous bars may be constituted one of equal resistance, we require:*

1st, that the k *geometrical relations, expressing that the alterations in length per unit of length, may be constant for all bars in tension and for all bars in compression may be satisfied, and 2d, that the resulting signs of the stresses must be agreeable to the laws of statics.*

If these conditions are satisfied for certain assumed sections, *the system will not cease to be of equal resistance, however we vary the sections, provided the resulting signs of the stresses are as first assumed; i. e., if a system containing superfluous lines can be constituted a system of equal resistance in one way, it can in an infinite number of ways.*

§ 7.

As it is a fundamental property of systems of equal resistance that the changes of length from strain, per unit of length is constant for bars in tension and for those in compression, we have a simple test to apply to any figure to see if it can be made a system of equal resistance. Thus, having assumed the bars elongated or compressed, according to the laws of statics, we have only to ascertain if, after deformation, the changes of length of all the bars in tension are the same per unit of length, and that the changes of length of all the bars compressed are the same per unit of length.

If this geometrical relation is fulfilled, then the system may be constituted one of equal resistance, otherwise it cannot, at least for the kind of strains assumed.

One case may be specially mentioned, where the bars are all supposed to be lengthened or all compressed, the same amounts per unit of length. The deformed figure is of course similar to the original figure, so that the first condition is realized, but the second is not, for such modes of deformation are not generally agreeable to the laws of statics. It will generally be found that most trusses with superfluous lines cannot be made of equal resistance. Thus we have seen in the case of the rectangle with two diagonals, that it cannot be so constituted, for the same unit stress throughout.

Let us examine Fig. 3 in this regard. First let us discard the upper bar, so that we have a figure formed of three bars, whose lengths are b_0, b_1 and b_2. Here we have one superfluous bar. Let us assume that the lower bar takes compression and the other two tension, which is agreeable to the laws of statics.

If the system is to be made one of equal resistance for tension and compression, the elongations per unit $\frac{f}{ew}$ must be the same for all three bars, so that Eq. (8) reduces to

$$-a_0^2+a_2^2=2a_1^2.$$

But as $a_2^2=a_0^2+4b^2$, and $a_1^2=a_0^2+b^2$, this reduces to

$$b=a_0;$$

so that if we construct the system so that this condition is satisfied, the bars will receive the same unit stress, no matter what sections are assumed, the stresses being varied to suit, provided the character of the stresses does not change.

In fact, for this case, when $b=a_0$ we can draw any stress diagram that will give the lower bar compression and the upper bars tension and proportion the sections for the same unit stress as assumed.

If the two lower bars are supposed compressed, we have as the necessary condition,

$$-a_0^2+a_2^2=-2a_1^2,$$

which reduces to $b=-\frac{a_0}{3}$, a negative solution indicating an impossibility.

Let us next test the original figures with four bars, and assume the three upper bars to take tension. Considering the relation between the three upper bars, $a_1^2+a_3^2=2a_2^2$, we deduce $2b^2=o$, an absurdity, as then the frame reduces to one line. If we assume the two lowest members compressive, the others tensile, eq. (8) in this case gives the absurdity found above, $a_0=-3b$, and eq. (7) the other absurdity $2a_0^2=o$.

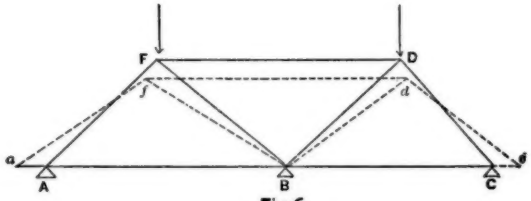


Fig. 6

Similarly, we could proceed on any hypothesis, agreeable to the laws of statics or a stress diagram. We see that for reasonable assumptions the system with four bars cannot be constituted of equal resistance, but that the system with three bars may be so constituted (by making $b=a_0$) in an infinite number of manners.

The same conclusions hold if the frame is turned upside down, only the corresponding stresses change character.

Let us next examine the continuous girder of two equal spans, Fig. 6, and see if it can be constituted a system of equal resistance. In this figure, the inclined members are all equal, and the sides AB, BC and FD are equal. We have here one superfluous line, say FD, since the joints F and D are fully ascertained when the sides of the two triangles ABF and BDC are given.

Under the influence of two equal vertical loads applied at F and D, the truss will be deformed to some other, shown by the dotted lines. If the superfluous bar FD was removed, both inclined members would take compression, and the members AB and BC would be extended, so that we shall make this first supposition for the full figure.

If we suppose the diagonals to be compressed an equal amount, and the sides AB and BC to be extended an equal amount in a horizontal direction (which involves the sliding on rollers say at A and C), then fd is horizontal and Bfdc and Bafd are parallelograms, since two opposite sides are equal, and the other two sides are parallel, so that fd=Ba =Bc, and the elongation of FD is the same as that of AB or BC. Now since these sides are equal in length, this is a necessary condition in order that the truss can be constituted a system of equal resistance, and as it is fulfilled we conclude that for the character of the stresses assumed, this truss may be made a system of equal resistance. The same holds when the truss is inverted, only the pieces formerly elongated are now compressed and the reverse.

If the truss was fixed at A, B and C, an equal compression of the inclined members would simply lower the apices F and D vertically, so that F D could receive no elongation, and the system cannot be constituted one of equal resistance, except when the bars AB, BC and FD are removed, when of course there would be no superfluous lines.

If we suppose AF and CD compressed and BF and BD elongated, it can easily be shown that the system cannot be made of equal resistance.

In fact if A and C are on rollers, it is evident that fd will be longer than Bc=Ba, since for the same height the triangle Bfd has one inclined side equal to one side of the triangles Baf or Bdc, and the other side longer, so that the base fd is longer than Ba or Bc.

This holds for a stronger reason if the joints A, B and C are all three immovable.

We have thus seen that for one combination of stresses, the system may be made of equal resistance. Further on, we shall resume this example again, and show that this combination is agreeable to the laws of statics.

For bridge trusses in especial, where rolling loads are concerned, constructors generally vary the unit stresses for the different members, so that there may be several values for c' and c'', eqs. (19) and (20), to satisfy. In such cases to ascertain if a certain truss can be constituted one of the varying resistances assumed, we suppose certain pieces compressed and others extended, agreeable to the laws of statics (and the simplest supposition would be that given by statics alone when the superfluous bars are omitted), and then ascertain if the alterations in length per unit of length, after elastic deformation are as assumed, regarding the geometrical connection of the parts.

§ 8.

When there are no superfluous bars, the frame can always be constituted one of equal resistance if desired, for statics furnishes at once the stresses in all the bars, irrespective of their sections, so that the last can be chosen at pleasure to suit the unit strains.

We have seen above that systems with superfluous lines cannot in general be constituted systems of equal resistance, but that when this happens in one way, they can be so constituted in an infinite number of ways by suitably varying the sections.

It is therefore pertinent to inquire, if such systsms with superfluous lines are not more economical than statically determined systems? If so, there is some justification in using them, otherwise not, even when they involve the same amount of material; for as misfits and other disturbing influences must occur in practice, the resulting stresses, for systems with superfluous lines will be different from the assumed, some being greater, some less; so that the limit of security is not as great as assumed; whereas in statically determined systems, the unavoidable misfits do not affect the strains, since each bar is free to change length, irrespective of the other bars, and the limit of security is the same as was assumed.

As preliminary to the inquiry before us, we shall establish the following lemma:

LEMMA.—*If a figure with* k *superfluous lines is such that we can, in one manner, and consequently in an infinite number of manners, dispose the sections of*

*its bars, so that it forms a system of equal resistance for the loading assumed, we can always, by suppressing some of the bars, form a system without superfluous lines, which subjected to the same loading, experiences the same elastic deformations as the primitive system, provided the signs of the stresses in the remaining bars do not change.**

Let us consider a figure formed of m bars, and containing k superfluous lines. We admit that it is possible in one and consequently in an infinite number of ways, by properly choosing the sections, to constitute it a system of equal resistance, so that all the bars in tension are extended an equal amount per unit of length, and all the bars compressed are shortened an equal amount per unit of length.

Let w be one of the sections of a superfluous bar satisfying the conditions.

Now if we decrease the section w of this bar (which changes its stress correspondingly), the stresses and sections of all the other bars will change. If the signs of the stresses in the other bars do not vary as we decrease w to zero, the system still remains one of equal resistance when $w=o$, or the bar in question is removed.

If, however, as we decrease w the sign of the stress in any other bar changes from + to − or the reverse, then for some value of w greater than zero, the stress in this other bar becomes zero and its section null, all the other stresses preserving their signs, so that with this bar removed, the system is again one of equal resistance. We can thus suppress one bar after another, until the system is freed of superfluous lines, provided the signs of the stresses of the remaining bars remain the same, and the system will still remain one of equal resistance. But for such systems, we have shown that the total changes of length of each bar remains the same, however we vary the sections, the signs of the stresses remaining unchanged, as happens in this case; *therefore the figure in question, after deformation, remains exactly the same, with or without the superfluous lines*, which proves the lemma as enunciated.

If the signs of the stresses change for the remaining bars, as we decrease in turn the sections of the superflous bars to zero, the figure of course no longer remains the same, after deformation, for the truss with and without superfluous bars. Levy has overlooked this important fact, which limits his following deductions to a very restricted class of figures. Thus the following theorem does not apply to continuous girders of many panels, braced arches fixed at the ends, &c., as Levy supposes; for on eliminating the superfluous bar or bars the character of the stresses in some of the remaining bars will generally change, and the elastic deformation is therefore not the same. In fact, for continuous girders the chords and web about the center piers are strained exactly in an opposite manner to what they are for single spans, except for the simple case given further on. If it is possible to eliminate some bar between the supports that will not change the character of the stresses of the balance, then the theory in question is applicable for such modifications.

* Levy does not assert the last saving clause in his enunciation.

§ 9.

As a consequence of the foregoing lemma, the sum of the *work* of all the exterior forces, applied at the joints, due to the elastic displacement of the joints is the same for the figure with or without superfluous lines for the case assumed. That is, this sum—call it T—is a constant.

Let t_i represent the positive tension of a bar, and a_i its elastic elongation; the work of the exterior forces developed in this bar, in consequence of the elastic displacements which produce the elongation a_i, is, from a well-known theorem of mechanics,

$$\tfrac{1}{2}\, t_i\, a_i\,.$$

Moreover, if the system is of equal resistance,

$$t_i = e_i\, w_i\, c',$$

whence

$$a_i = a_i \frac{t_i}{e_i\, w_i} = a_i\, c',$$

and,

$$\tfrac{1}{2}\, t_i\, a_i = \frac{c'^2}{2}\, e_i\, a_i\, w_i\,;$$

consequently, the sum of the work of the elastic forces of all the bars which are elongated, is

$$\tfrac{1}{2} \Sigma t_i a_i = \frac{c'^2}{2} \Sigma a_i e_i w_i .$$

The stress of a bar which is compressed, is

$$t_j = - e_j w_j c'',$$

its elongation,

$$\hat{a}_j = - a_j c'' ;$$

whence for the work of compression, we have

$$\tfrac{1}{2} t_j a_j = \frac{c''}{2} a_j e_j w_j ,$$

and for the sum of the work of the compressions,

$$\frac{c''}{2} \Sigma a_j e_j w_j .$$

The sum of the work of all the elastic forces of the system, tensions and compressions, is then

$$\frac{c'}{2} \Sigma a_i e_i w_i + \frac{c''}{2} \Sigma a_j e_j w_j = \mathrm{T},$$

which sum is necessarily equal to the work T of the exterior forces.

If we regard the material as resisting tension and compression equally well, so that $c'=c''$, this equation becomes, regarding Σ as extending to all the bars, whether in tension or compression,

$$\frac{c'^2}{2} \Sigma aew = \mathrm{T} \quad . \quad . \quad . \quad . \quad . \quad (24).$$

If we assume that all the bars have the same modulus of elasticity e, this equation becomes

$$\Sigma aw = \frac{2\mathrm{T}}{ec'^2} \quad . \quad . \quad . \quad . \quad (25).$$

The product aw is the volume of the bar of the length a; the first number represents then the total volume of material employed, and as the second member is the same, for the system with as without the superfluous lines, we conclude:

THEOREM III.— *When a system containing* k, *superfluous lines, is such that it can in one manner, and consequently in an infinite number of manners, be constituted a system of equal resistance, having the same unit stress for each bar, for a given loading, there exists always a system without superfluous lines, capable of resisting the same external forces and employing only the same amount of material, provided the bars belonging to both systems retain the same kind of stress, however we vary the sections of the superfluous bars towards zero.*

Thus, in this particular case, where we can, without ceasing to employ the same unit stress, employ figures with superfluous lines, there is no economy in doing so, at least for the loading assumed.

If the bars have different coefficients of elasticity, we see from Eq. (24) that the last theorem can be replaced by the following:

THEOREM IV.— *When a system containing* k *superfluous lines is such that it can, in one manner, and consequently in an infinite number of manners, by suitably choosing the sections, be constituted a system of equal resistance, for given external forces, there always exists a system without superfluous bars, capable of withstanding the same forces with the same unit stress as before, such that the sum of the products of the volume of the bars by their coefficients of elasticity is the same in this system and the given system for the special case where the character of the stresses in the bars remains the same for the system with or without superfluous bars.*

Now as the sum of the products above represents in some sort the *elastic weight* of all bars, we see that here, as in the preceding case, that it is not advisable even when we can, to use figures with superfluous lines, if the truss is to be proportioned only for the given case of loading.

These are remarkable theorems, not only on account of the simplicity of the demonstrations, but mainly because of the generality of the conclusions. *It applies to every form of roof truss, trestle piers, etc., or any structure whatsoever, whose parts are proportioned to resist the same unit stress for one kind of loading* and stress in accordance with the hypothesis.

They prove beyond all question, for such structures, that the system without superfluous bars is at least as economical as when they are added.

The theorems likewise apply to bridge trusses that are designed for one position of the applied load, as in aqueduct bridges and in some highway bridges. For these structures, designed as stated,

there is no economy in the use of any form of truss whatsoever that has more lines than are strictly necessary to construct it geometrically.

So we conclude that, even when bridge trusses with superfluous bars, designed for one method of loading and stress, *can be made* systems of equal resistance, which moreover rarely happens, there is no economy in their use if the superfluous bars may be eliminated without changing the kind of stress of the remaining bars, even when we leave out of consideration the very great influence of misfits and the effects of settling of the piers and abutments, &c.

In railroad bridges, and many highway bridges as designed by some engineers, we no longer make the system one of equal resistance for one position of the live load, but proportion the members of the truss for the maximum stresses that may be caused by any position of the live load, so that Levy's theorem no longer applies to such bridges.

§ 10.

It may not be amiss to examine the two cases of systems of equal resistance already found in relation to Levy's theorem, that the amount of material remains the same however we modify the sections, as they afford a striking illustration of the theorem in question and are moreover very easily treated.

In the case of Fig. 3 with the top bar omitted, equations (5) and (6) reduce to the following, when the two top bars are supposed to take tension and the bottom bar compression which, it has been shown, constitutes this a system of equal resistance when $b=a_0$,

$$f'_1\frac{b}{a_1}+f'_2\frac{2b}{a_2}=\mathrm{W} \quad \ldots \quad (26)$$

$$-f_0+f'_1\frac{a_0}{a_1}+f'_2\frac{a_0}{a_2}=o \quad \ldots \quad (27).$$

Compression and tension are both plus in these equations.

On dividing these equations by the common unit stress s, and reducing we get the following relations between the sections:

$$w_1a_2b+w_2.2a_1b=a_1a_2\frac{\mathrm{W}}{s} \quad \ldots \quad (28).$$

$$w_0a_1a_2-w_1a_0a_2-w_2a_0a_1=o \quad \ldots \quad (29).$$

If we call M the volume of the material,

$$w_0a_0+w_1a_1+w_2a_2=\mathrm{M} \quad \ldots \quad (30)$$

On multiplying (30) by a_1a_2 and (29) by (a_0), and subtracting the latter from the former, we have

$$\mathrm{M}a_1a_2=w_1a_2(a_1^2+a_0^2)+w_2a_1(a_2^2+a_0^2)$$

Or reducing, since $b=a_0$, $(a_1^2+a_0^2)=3b^2$, and $(a_2^2+a_0^2)=6b^2$,

$$\mathrm{M}a_1a_2=3b[w_1a_2b+w_2a_1 2b].$$

Or since the quantity in the brackets equals (28), we have

$$\mathrm{M}=\frac{3b}{s}\mathrm{W}=a\ constant,$$

or the material is the same however we vary the sections according to laws previously established; so that we can diminish the section of one of the upper bars to zero, and the *resulting* volume of the remaining two bars remains exactly the same as for the three bars, both systems being of equal resistance, and subjected to the same kind of stress. Mr. Emil Adler, C. E., has kindly communicated the foregoing result, as well as the one pertaining to the next case, though his method of demonstration is independent in many respects of the one followed here.

Let us next consider the very simple case of a continuous girder of two spans like Fig. 6 or Fig. 7 below, in which the figure is made up of isosceles triangles, and the equal loads are applied at the upper apices. We have seen that this system can be made one of equal resistance if the inclined members all take stress of one kind and the horizontal members stress of the opposite kind, provided this supposition is agreeable to the laws of statics.

Call the equal length of the inclined members a, and the length of either span which equals the length of the top member l, and the height of truss h. The stresses in the bars will be as designated in Fig. 7. In consequence of symmetry, the stresses in corresponding members, either side of the center are equal. The equal unknown reactions at the end supports will be called nP, whence the reaction of the middle support is $2\mathrm{P}(1-n)$.

Now regarding tension and compression as both plus we have for the inclined members in the compression and the others in tension,

$$f_1=\frac{nP}{h}a$$

$$f_2=\frac{P}{h}a\,(1-n)$$

$$f_3=\frac{nP}{h}\,\frac{l}{2}$$

$$f_4=\frac{P}{h}\,\frac{l}{2}(1-2n)$$

having one superfluous bar, we can choose at pleasure the section of any one bar, which involves its stress likewise, and by the aid of the 6 statical equations above (two each for f'_1 and f'_2) determine the stresses and afterwards the sections of the other bars. Thus if we assume the section w_1 of the outer inclined member, whence the stress on it is found from the eq., $f'_1=cew_1$, we thereby determine the reaction nP from the first equation above; or we may assume nP and compute f'_1 from this equation and thus determine all the other stresses from the

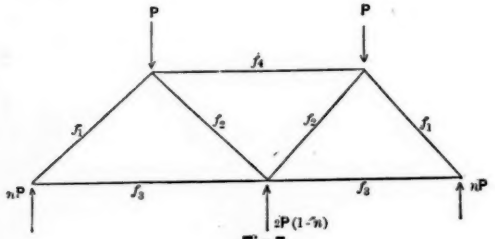


Fig. 7

These stresses are all plus as assumed, so long as $n \leqq \frac{1}{2}$, as we see from the last equation, move particularly; so that when $n \leqq \frac{1}{2}$, the system may be made of equal resistance.

On dividing each stress by the assumed unit stress s, multiplying by the length and adding the results, we obtain for the total amount of material for the entire truss, both spans,

$$\frac{P}{sh}\left(2na^2+2a^2(1-n)+nl^2+\frac{l^2}{2}-nl^2\right)$$

$$=\frac{P}{sh}\left(2a^2+\frac{l^2}{2}\right)$$

This result is independent of n, so that the amount of material remains the same however we choose n, provided we do not exceed the limits o and $\frac{1}{2}$. Thus we see that Levy's theory is verified for these two cases of systems with superfluous bars, as indeed it must be for all cases, as it rests upon a strict mathematical basis.

In the case of the last truss, Fig. 7, group of equations. So that we are conducted to an interesting property of this truss, *that if we assume the reaction at pleasure between easily appreciated limits, deduce the stresses, and design the sections accordingly for the same unit stress, that the assumed reaction will be the actual reaction resulting from the sections assumed.*

Mr. Adler first called my attention to this principle, demonstrating it in a different and more elaborate manner. If we make the end reaction zero, the end braces and lower chord disappears, as we see from the first and third equations above. Again if we assume $n=\frac{1}{2}$, the stress $f_4=o$, and the figure reduces to two discontinuous spans. As shown above, we have therefore for the same loading, the same amount of material in the three trusses, shown in Fig. 8.

We see how marked the influence of the web is in this example, for by varying the section of the end brace, which involves a corresponding alteration in all the sections, we can cause the reaction to vary from o to $\frac{1}{2}$ P at pleasure, and the continuous girder reduces to the simple bracket, or to two continuous spans at the respective limits.

§ 11.

We shall next examine a simple form of roof truss (Fig. 9) given by Bow ("Economics of Construction," p. 84), especially for the case of an invariable span, though we shall compare stresses on the three different suppositions of truss on rollers at supports with and without a horizontal bar, and for truss fixed at supports or span invariable.

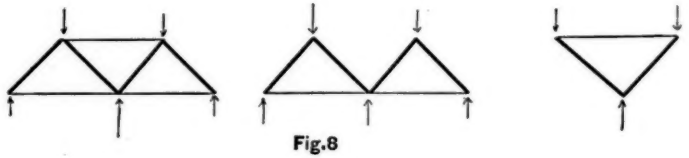


Fig.8

There are 4 joints in this truss and 6 bars, so that $m>2n-3$, and there is one superfluous bar.

Denote the lengths of the bars AB, AD AC and DC by $2a_1$, a_2, a_3 and a_4 respectively and their corresponding stresses, sections and moduli by f_1, w_1, e_1; f_2, w_2, e_2; f_3, w_3 e_3; f_4, w_4, e_4, respectively; also call h the height of point D above the horizontal bar AB.

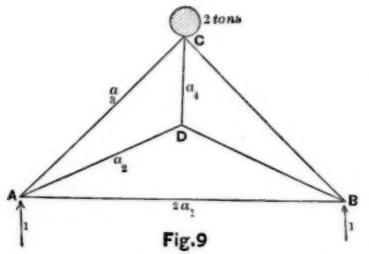


Fig.9

We have the evident relation between the lengths,

$$F=\sqrt{a_3^2-a_1^2}-\sqrt{a_2^2-a_1^2}-a_4=0 \ldots (1).$$

From § 4, we draw the following equations, a_1, a_2, a_3, a_4, represent the elastic elongations of bars AB, AD, AC and CD after strain:

$$\frac{dF}{da_1}a_1+\frac{dF}{da_2}a_2+\frac{dF}{da_3}a_3+\frac{dF}{da_4}a_4=0 \quad \ldots (2)$$

$$\frac{dF}{da_1}a_1\frac{f_1}{e_1w_1}+\frac{dF}{da_2}a_2\frac{f_2}{e_2w_2}+\frac{dF}{da_3}a_3\frac{f_3}{e_3w_3}+\frac{dF}{da_4}a_4\frac{f_4}{e_4w_4}=0 \ldots (3).$$

On differentiating (1) with respect to a_1, a_2, . . . successively substituting in (3) and reducing, we obtain,

$$(\sqrt{a_3^2-a_1^2}-\sqrt{a_2^2-a_1^2})a_1^2\frac{f_1}{e_1w_1}$$
$$-\sqrt{a_3^2-a_1^2}.a_2^2\frac{f_2}{e_2w_2}+\sqrt{a_2^2-a_1^2}.a_3^2\frac{f_3}{e_3w_3}$$
$$-\sqrt{a_3^2-a_1^2}\sqrt{a_2^2-a_1^2}.a_4\frac{f_4}{e_4w_4}=0,$$

which reduces to

$$a_4a_1^2\frac{f_1}{e_1w_1}-(h+a_4)a_2^2\frac{f_2}{e_2w_2}+ha_3^2\frac{f_3}{e_3w_3}$$
$$-(h+a_4)ha_4\frac{f_4}{e_4w_4}=0 \ldots . (4).$$

We may first inquire if the system can be made of equal resistance, so that

$$\frac{f_1}{e_1w_1}=\frac{f_2}{e_2w_2}=\&c.$$

On substituting and reducing, we get for the hypothesis that all bars are compressed or all extended, the identity, $o=o$, as we should § 7; but such a supposition is not agreeable to the laws of statics.

If we suppose all the bars in compression except AB, so that f_2, f_3 and f_4 are minus, and f_1 plus eq. (4), reduces to the absurdity, $a_4a_1^2=o$. Similarly the supposition that f_3 is minus (compression) and f_1, f_2 and f_4 plus (tension) causes an absurdity.

These are the only reasonable suppositions that can be made as a stress diagram will show; so that the system cannot be made one of equal resistance.

Let us now ascertain the stresses in the frame for a weight of 2 tons resting on the summit, the ends of the truss being free to move, on imaginary perfect rollers, the lengths being taken as follows: $a_1=1{,}000$. $a_2=1{,}118$, $a_3=1{,}414$, $a_4=500$ and $h=500$, *and the sections of*

all the 3 bars being taken the same, as well as their moduli.

We must frame our equations upon the supposition (already made in 4) that all the bars are in tension, so that the forces due to the bars act away from any joint considered. The reactions are, of course, 1 ton each and act upwards. Upon solving the equations, we of course find the proper signs for the stresses, plus corresponding to tension, and minus to compression. Expressing now that the sum of the horizontal and vertical components of the forces acting at A are separately equal to zero, balancing the vertical components of the stresses in the bars meeting at D, and substituting the numerical values in (4), we get the four following equations to determine the stresses in the four bars.

$$1+\frac{500}{1118}f_2+\frac{1000}{1414}f_3=0 \quad \ldots . \quad (5)$$

$$f_1+\frac{1000}{1118}f_2+\frac{1000}{1414}f_3=0 \quad \ldots . \quad (6)$$

$$2\frac{500}{1118}f_2-f_4=0 \quad \ldots . \quad (7)$$

$$2f_1-5f_2+4f_3-f_4=0 \quad \ldots . \quad (8)$$

By successive elimination, we deduce the following numerical values:

$f_1=+1.17$ tons (tension).
$f_2=-0.39$ " (compression).
$f_3=-1.17$ " "
$f_4=-0.35$ " "

As the sections were taken equal, the unit strains for all the bars varies as the stresses above.

Let us next ascertain the stresses for *span invariable*, so that a_1 reduces to zero in Eq. (2) whence (4) becomes for *equal sections and moduli*

$$-5f_2+4f_3-f_4=0. \quad \ldots . \quad (9).$$

The horizontal bar AB must now be removed, as it cannot change length, and consequently cannot suffer strain, and we shall suppose horizontal forces f_1 acting inwards at A and B to represent the horizontal components of the reactions, the vertical components remaining as before.

Equations (5), (6) and (7), under this supposition, still hold, so that from these equations and Eq. (9), we are to find by elimination, the stresses, which are as follows:

$$f_1=+1.306 \text{ tons.}$$

Acting inwards as assumed, otherwise the sign would be different,

$f_2=-0.67$ tons (compression.)
$f_3=-1.00$ " "
$f_4=-0.6$ " "

On combining the vertical component of the reaction = 1 ton, with the horizontal component = 1,306 tons, we find the resultant reaction, whose position thus lies between that of the two inclined bars AC and AD.

We may next find the stresses for this truss with the horizontal bar left out, supposing the truss to rest on frictionless rollers, so that the reactions are vertical. The stresses as found from a diagram are:

$f_2=+2.236, f_3=-2.828$ and $f_4=+2$. tons.

Let us tabulate these results, for a weight, resting at the summit, of 1,000, so that the changes the stresses undergo for the different suppositions may be seen at a glance:

Truss on rollers. Bar AB removed.	Span invariable. Bar AB removed.	Truss on rollers. Bar AB retained.
		$f_1=+585$
$f_2=+1118$	$f_2=-335$	$f_2=-195$
$f_3=-1414$	$f_3=-500$	$f_3=-585$
$f_4=+1000$	$f_4=-300$	$f_4=-175$

In the first case, where statics alone determines the stresses in the bars, we can suit the sections to any unit strain, but in the other two cases the sections were all supposed equal and the *unit strains* vary very greatly. We have just seen that it is impossible to make them equal, for this truss; but we may approximate nearer to this result by choosing different sections and computing strains and so on; in other words, by a laborious tentative solution.

This example will give some idea of how strains are materially affected by the condition of the supports, which are probably never exactly as assumed.

In this connection we shall give Bow's method of finding the reactions and

stresses for any assumption regarding the yielding of the support, or for span invariable. This method is in brief "to assume in succession two different directions for the reaction of the abutment and calculate for each the change caused in the length of the span; the reaction or supporting force that will cause no change of length in the span is then easily ascertained by taking for its components such proportions of the two assumed reactions, that their effects in altering the length of the span will neutralize one the other." If a certain change of span is assumed, the reactions could be found in a similar manner.

Assuming the weight resting on the summit as 1,000, Bow finds, for the reaction vertical and equal to 500, the change of span = + 6.1, and for two horizontal forces, acting inwards at both abutments, each equal to 500, the change of span, −4.7; so that the ratio of the true horizontal component to the vertical reaction to cause no change of span is $\frac{6.1}{4.7}=1.3$, which agrees with what we have found above in an entirely different manner.

Bow does not state how these changes of span are computed, but we readily see that it may be effected by aid of eq. (2) above, or for this particular example from (4) modified as below:

$$a_4a_1a_1-(h+a_4)a_2^2\frac{f_2}{e_2w_2}+ha_3^2\frac{f_3}{e_3w_3}$$
$$-(h+a_4)ha_4\frac{f_4}{e_4w_4}=0$$

As we only desire relative changes of span, we can put

$$e_2w_2=e_3w_3=e_4w_4=100,$$

for ease of computation, so that the above equation becomes, on substituting numerical values,

$$a_1=25f_2+20f_3-5f_4.$$

By aid of a stress diagram, we find for reactions vertical,

$$f_2=+1118,\ f_3=+1414,$$

and

$$f_4=+1000,$$

whence

$$a_1=61{,}230.$$

For the truss subjected only to the two horizontal forces, taken equal to the vertical reactions just mentioned, we find

$$f_2=-1118,\ f_3=+707,$$

and

$$f_4=-1000,$$

whence

$$a_1=-47{,}090,$$

so that the ratio of the horizontal to the vertical component of the reaction is,

$$\frac{61{,}230}{47{,}090}=1.3,\text{ as found above.}$$

This method may be preferred in some cases to the preceding, and in all cases should be used as a check.

§ 12.

We have now given the general method to be followed in treating frames with superfluous bars, and illustrated the subject by some of the simpler examples. The solution becomes more and more complex as the number of members of the frame increases, besides it is generally impossible to constitute trusses, having many subdivisions, systems of equal resistance, even for one given case of loading. American engineers generally have wisely avoided such systems and restricted themselves in practice to trusses whose parts can be computed by the simple laws of statics and that can be made systems of equal resistance, if desired, or whose parts can separately be subjected to any unit stresses that experience has approved. Thus most of our roof trusses can be statically determined; also the single intersection bridges as the Pratt and Howe types; for it has been shown (§ 5) that the counters (which are superfluous bars, if in action at the same time as the main diagonals) are not in action when the corresponding main diagonals are in action and *vice versa, so that the number of bars* (m) *under stress at the same time remains constant and equal to,* $2n-3$, where $n=$ number of joints, as may be readily verified.

The same relation, $m=2n-3$, will be found to hold for the Warren girder and modifications, the bow string, Schwedler and other single intersection systems, and systems whose diagonals are not crossed and which can take compression and tension both for certain panels. The Fink truss, too, will be found to be stati-

cally determined, as well as the Bollman when the panel diagonals are left out.

But for double intersection bridges it seems impossible to prove in some cases that *the number of bars under stress remains constant* for any loading and equal to $2n-3$, or the number of bars when the counters are omitted; so that the common supposition to that effect is not strictly accurate.

Thus in the double intersection quadrangular deck truss below (Fig. 10), where the two partial systems into which the truss is supposed divided, are marked with heavy and light lines respectively, let us suppose a live load to extend from the right abutment to joint 7, and that counter G5 is in action, which consequently throws E7 out of action, similarly E3 is in

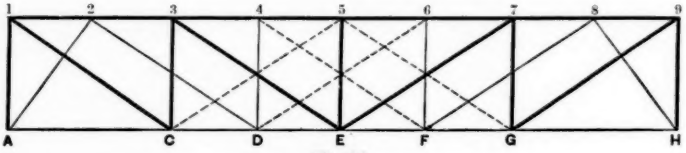


Fig. 10

action and C5 out of action, so that the total number of bars under stress in the one partial system shown by the heavy lines remains the same. But on considering the other partial truss, the dead load at F may go partly by F8 to right abutment and partly by F4 to left abutment. If F4 is strained, D6 is not; still if F4 and F8 are both strained at the same time, the truss will be found to have one superfluous bar, so that it is statically undetermined for this particular loading; for the number of bars is now 30, and the number of joints 16, so that m exceeds $(2n-3)$ by one.

The common supposition is that the dead load at F goes to right abutment, but it is unproved and is incorrect if the greatest deflection of the truss is at G, for then all diagonals to the left of G, parallel to G5 are under tension and the diagonals crossing them are shortened and thus out of action; so that under this supposition F4 is in action and F8 out of action. There are thus two horns to the dilemma, either the system may be statically undetermined or the common theory is not strictly correct. The most correct solution consists in finding the reaction at A for the whole truss, and subtracting the loads on one system up to the point of greatest deflection to get the reaction for the other partial system; but as we cannot fix this point of greatest deflection the indetermination still exists. The difference between the true and common methods is probably slight, for well-fitting trusses with counters properly adjusted, and the method in vogue is likely on the side of safety; still it is to be regretted for this popular form of truss that any indetermination should exist as to the stress in the members.

It might be thought that a trellis bridge, whose diagonals can take tension and compression both, was free from the defects of the preceding truss, but we shall not find it so. In fact for a trellis truss of eight panels, we have, $m=30$ and $n=16$, so that $m>2n-3=29$, and the system is statically undetermined.

It may not be amiss to notice here an opinion entertained by some, that a misfit in a diagonal eye bar say, would cause extra strains over those computed equal to the force required to stretch the bar to its calculated length, which may amount to several tons strain to the square inch. It is hoped that the foregoing discussion has demonstrated that for statically determined systems, with joints free to move, that the usual misfits has no influence on the strains. If the joints are not free to move, as in the upper joints of some bridges, or if the system has superfluous bars, the strains are not as computed, but even then there is no simple relation like the above to ascertain the extra strains. It is known that even with pin connected bridges, there may be sufficient friction at the joints or imperfect action of the rollers to disturb the strains given by statics alone on the supposition of perfectly free joints; but leaving this to one side, it is evident that as pin connected trusses, without super-

fluous bars, can be corbelled out piece by piece from one end, as was done in the Kentucky River Bridge (C. S. Ry.) that every piece must come to its bearing, and there can be no extra strains from misfits that are appreciable.

§ 13.

FRAMED PIERS.

Framed piers and trestle bents have often either a lack or a redundancy of parts or both, so that the stresses in them cannot be determined by statics alone, except perhaps for a uniform vertical loading.

Of late much more attention has been given to wind pressure on piers than formerly, resulting in simple forms that statics can handle. It will be the principal object of this section to treat such forms fully (especially as, so far as the writer knows, they have never received a thorough and accurate analysis), as well as to discuss other well-known designs with a view principally to pointing out their defects and of analyzing some of them.

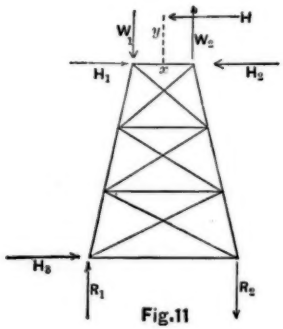


Fig. 11

Let Figs. 11 and 12 represent one bent of a framed pier, subjected to the total wind force H on trusses and train, sustained by it, acting at its center of pressure, a distance y above (Fig. 11) or below (Fig. 12) the top of the bent.

Where the pier sustains a through bridge or a deck bridge supported at the lower chord, H will always act above the pier, though it may happen otherwise when the pier sustains a deck bridge swung from the top chord. In the latter case the component of H, supposed concentrated on the lower chord will act below, whilst the components acting on the upper chord and car surface, will act above the upper member of the pier.

The position of H can readily be found by equating the sum of the moments of the wind pressure acting on the upper and lower chords and car surface about the top of the pier with Hy, giving, say a positive sign to a left-handed moment, and a negative sign to a right-handed moment. The resulting sign of y will show whether H acts above or below the top of pier.

If we add now the two equal and opposed forces, H_1, H_2, acting along the top member, whose length is x, we do not disturb equilibrium, but the single force H is now replaced by the couple HH_1, and the single force H_1 acting against a member that can sustain it.

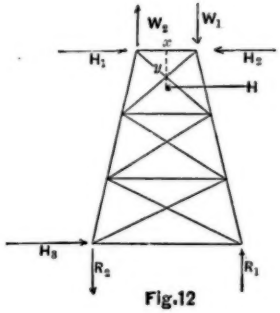


Fig. 12

Now if the equal vertical forces W_1 and W_2, acting in opposite directions at the tops of the inclined columns where they can be sustained, are of such a magnitude and direction that,

$$W_1x = W_2x = Hy,$$

then the couple W_1W_2 can replace the equal couple HH_1; so that we have finally as the equivalent of H_1 the forces H_2, W_1, W_2, all acting along members capable of sustaining them.

In Fig. 11 as HH_1 and consequently W_1W_2, are left-handed couples, W_1 acts to increase the weight on the leeward column and W_2 to decrease the weight on the windward column. The reverse obtains for Fig. 12.

The reactions R_1 and R_2 are readily obtained by equating the moment of the couple R_1R_2 with that of the couple $HH_2 = H \times$ height above lower sill.

The reactions are however readily found from a stress diagram without any computation whatever.

Generally these framed piers consist of two bents braced together, so that the total wind force on one bent is one-half that on the trusses and train on the adjacent span. The same holds for the outside bents, where the pier is composed of any number of bents braced together, though in this case the other bents will materially assist if overturning of the outside bents is about to take place. Still it is proper to design these outside bents on the supposition that they receive no aid whatsoever from the interior bents, especially if none of the columns are to be subjected to tension which is ordinarily good practice.

When the pier consists of but one bent only, the wind force on it is that caused by the wind acting on one-half of trusses and train on both adjacent spans.

Exactly the same relations hold as to the *weight* of trusses and train sustained by bents, disregarding the wind, so that it is very easy to compute for trusses loaded or unloaded the total resultant vertical forces at top of columns, as well as the horizontal force H due both to the weight of trusses and train and to the wind acting on them.

We shall suppose this done in what follows.

In the following figures one set of diagonals are left out, since the truss, on distortion sideways, will bring one set into action only, as these diagonals are usually made of bars of such small section that they cannot take an appreciable compression.

§ 14.

Having found, as just shown, the forces W_1, W_2 and H_2 due to wind force alone, and added, with the proper signs, the vertical loads due to the weights sustained by the pier, we can now proceed to draw the stress diagram Fig. 13 (*b*). Bow's admirable notation is used by which a bar or a force, in Fig. 12 (*a*), is designated by the letters between which it is placed and the stress on the bar or the magnitude of the external force is shown, in Fig. 12 (*b*) by the lines, to scale, at whose ends are the same letters. Thus the external forces due to the weight of the trusses and train and the wind force acting on them are given in position in Fig. 12 (*a*) by GH, HI and LG, and in magnitude, to scale, in Fig. 12 (*b*) by the corresponding lines GH, HI and LG. The reactions are similarly represented by JI, KJ and KL. In (*b*) these forces taken in any order and true direction, should form a closed polygon LGHIJKL, as obtains here.

On drawing in (*b*) the sides HA, AB, . . ., parallel to HA, AB, . . ., in (*a*), we form the stress diagram in which the stress in any member as AB in (*a*) is given to scale by line AB in (*b*). These stresses

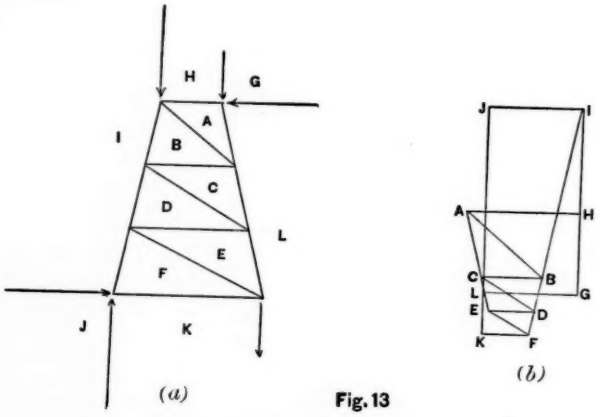


Fig. 13

are tensile or compressive, as in following around the polygon for each joint in the proper order, the force acts away from or towards the joint considered.

In this figure, for the proportions and forces given KL is a downward reaction, so that a holding down bolt is requisite.

As a rule, American engineers give sufficient spread to the base, so that no tension is exerted in the windward column.

We notice here, as in the next figure, that on constructing the stress diagram, beginning at the top of the pier, we find the reactions without any computation, though they may be tested as well as any of the stresses by the method of moments.

an able paper by C. Shaler Smith, and discussions thereon in the transactions of the American Society of Civil Engineers, for Dec. 15, 1880, and republished in *Engineering News* for Oct. 1, 1881.

Mr. Smith gives the following specification for piers: "Iron piers and spans carried by them shall be designed to resist a wind force of 30 lbs. per square foot on train and structure, or 50 lbs. per square foot on the structure alone.

"The compressive strains on the leeward columns of the piers shall be computed with the assumption that the maximum load is on the bridge, and to these shall be added the compressive strains produced by the wind, and the columns shall be proportioned to resist these com-

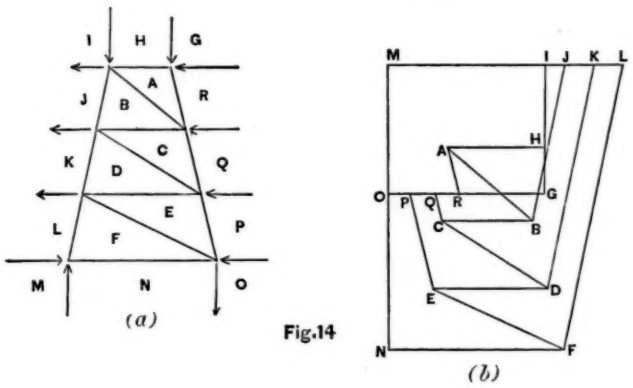


Fig. 14

If the pier is of such a height that it would be unsafe to neglect the force of the wind blowing directly on it, we must ascertain the horizontal wind force acting directly at each apex, when the stresses are quickly found from the following diadram, Fig. 14 (*b*).*

The force polygon here which is closed, as it should be, is Fig. 14 (*b*),

GRQPONMLKJIHG.

We shall find for this figure, that a holding down bolt is necessary and that the segments EP and CQ of the windward column are in tension, AR being in compression, whilst for the previous figure, the lower segment LE is in tension.

As to the amounts of wind pressure per square foot allowed in practice, see

bined strains with a factor of safety of four. The minus strains on the windward column shall be computed with the lightest train on the bridge, which will not be blown off by a wind force of 30 lbs. per square foot, and such a width of base shall be given to the pier that there shall be no tension in any of the columns composing it."

The pressure of 30 lbs. per square foot was specified principally because empty cars are blown over at that pressure. A higher pressure than 50 lbs. on the structure alone has been advocated by some engineers. It is evident, too, that in some situations it may be well to design the pier to resist tension in the windward column for the maximum wind pressure, but as a rule it is not advantageous.

The form of truss given in the preceding figures, without superfluous bars, is

* The weight of pier is similarly included in any stress diagram, by combining the proper weight at each apex with the wind pressure.

that most generally adopted now for iron piers, and no better can well be devised either for single or double track railways.

The piers for Mr. Shaler Smith's Kentucky River Bridge are of this form, only the tops of the two bents are drawn together and vertical struts extending from the bottom upwards to the first horizontal member below the top one, give a support merely at the middle of these horizontal compression members, which does not disturb the strains as given by statics alone.

§ 15.

A form of pier, shown by the following figure, only with two sets of diagonals, in place of one as shown, has been proposed for double track railways; but the system is faulty in having superfluous lines.

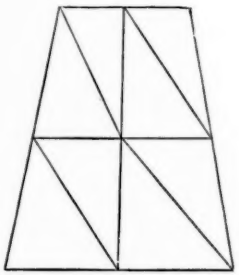


Fig.15

Thus for the number of divisions shown, we have $n=9$ and $m=16 \therefore 16>18-3=15$, and we have one superfluous bar. Therefore to compute the strains arising from any given loading we must write that at each joint the sum of the horizontal components of the forces, including the stresses, are zero, and that the sum of the vertical components are zero. This gives 18 equations or 15 independent ones. The additional equation is found by considering any one of the right triangles, whose hypothenuse has a length a_1 and the other sides the lengths a_2 and a_3 respectively, so that we have the relation,

$$a_1^2=a_2^2+a_3^2,$$

whence, as previously explained, we derive,

$$a_1 a_1=a_2 a_2+a_3 a_3,$$

and,

$$a_1^2\frac{f_1}{e_1 w_1}=a_2^2\frac{f_2}{e_2 w_2}+a_3^2\frac{f_3}{e_3 w_3}.$$

This last equation added to the others furnished by statics gives 16 equations to determine the stresses in the 16 bars.

The next figure (16) has the main compression members in the shape of an inverted W, and suffers even more than the previous truss from superfluous bars. Half of the diagonals are supposed out of action from the side pressure; but, even then, we have, $m=17$ and $n=9$, so that we have, $m-(2n-3)=17-15=2$ snperfluous bars.

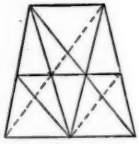


Fig.16

Consequently to the 15 independent equations of statics we must add two equations resulting from the geometrical relations between the sides. Thus consider one of the triangles above, whose sides have the lengths, a_1, a_2, a_3, respectively; the acute angle formed by the sides a_2 and a_3 being desigated by θ, we have the well known relation,

$$F=a_1^2-a_2^2-a_3^2+2a_2a_3.\cos\theta=0.$$

On giving the sides the increments in length a_1, a_2, a_3, and subtracting the first equation from the second, neglecting differences of the second order, we obtain,

$$a_1 a_1-(a_2-a_3\cos\theta)a_2-(a_3-a_2\cos\theta)a_3=0.$$

On substituting the values for $a=\frac{f'a}{ew}$ we obtain one of the required equations. Similarly the other is obtained by considering one of the other triangles, so that as many equations as bars can be written and the stresses in those bars determined by elimination between the 17 equations.

The labor of ascertaining the stresses even for as few divisions as we have taken is very great and is enormously increased for high piers with many subdivisions of the columns. It is more than probable that with the column material concen-

trated in the two outer braces, that not only would the strains be easily determined, but the pier would be materially lighter. At any rate, it is doubtful if a correct method of calculation has ever before been applied to these piers with superfluous members, so that no correct comparison has been made between them; but the writer is far from recommending them, even if they should show economy, as the strains are subject to such wide alterations from misfits, settlements of the foundations, heat of the sun on one side, etc., that any apparent economy is not real and only misleading.

§ 16.

The forms so far considered are about the simplest that have been used in iron construction. The more complicated forms are objectionable in so many respects that they should unhesitatingly be condemned. Of such objectionable types are those piers whose bases are not rectangular as we have assumed hitherto, but six or eight sided; so that the whole pier with its internal bracing must be considered if any attempt is made to design them scientifically. The six sided base, shaped like an ordinary masonry pier with cutwaters, is about as bad a design as could well be imagined where wind pressure is concerned. It is no wonder that the Tay Bridge piers, which were of this design, failed when subjected to a strong side wind with the train passing at the time, probably from weakness of the internal bracing which was designed by some rule of thumb method. To compute the strains in such a structure, it would be necessary at each joint to express that the components of the forces and stresses in the directions of three rectangular axes were separately equal to zero. As there are six necessary relations between the external forces, the number of independent equations reduces to $3n-6$. To these equations we must add $m-(3n-6)$ equations derived from the geometrical relations of the sides, from all of which the resulting stresses for *assumed* sections can be found and the unit stresses determined.

§ 17.

We shall conclude this discussion by a consideration of trestle work and trestle piers in wood, which can offer but little tensile resistance at the mortise joints, so that we can safely assume that certain pieces that would otherwise be in tension, are out of action and the computations because very much simplified and possible in some cases by statics alone.

The adjoining figure gives a skeleton outline of the most common trestle bent

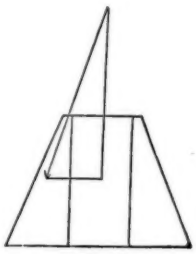


Fig.17

in wood, with posts vertical and braces inclined from 3 to 5 inches per foot. If the weight of bent is neglected, we simply combine weight of cars for distance between bents, center to center, with the horizontal force of wind acting at its center of pressure to get the resultant shown by the arrow acting at the cap sill. If this falls between a post and brace as shown, as will happen ordinarily for a 30 lb. pressure on empty cars when the batter of the brace is 5 to 12, the bent is stable and the whole weight is sustained by these two columns. If the posts are spread further apart so that this resultant passes between the posts the vertical component is divided between them according to the law of the lever, but the horizontal component acting along the cap must all be sustained at the left, as the right brace and post cannot receive tension; so that at the intersection of the center lines of the post and brace (when not far apart) we recombine the horizontal component or thrust of wind with the vertical load sustained at the left post for the total resultant, which may then be decomposed following the center lines of post and brace. If this resultant passes outside the brace the bent is unstable as the post cannot receive tension.

For this form in iron, the horizontal component acting along the cap is sustained equally at the two apices, since

any horizontal movement of the cap affects both to the same amount, and since the figures formed by the post brace and base sill are similar, right and left, the small deformation and resulting strains are the same for both figures.

Where the weight of bent and track is considered, in addition to the weight of train and the side pressure of the wind, we simply combine the weight of bent and track sustained at a post with the vertical components of the train and wind load conveyed there, for the total vertical component, which combined with the wind pressure gives the resultant required.

For the usual sizes of timbers and bents 12½ feet from center to center, we find that a batter of 4 to 12 will ensure sufficient stability, and even less may be used though it is not advisable.

§ 18.

This form is not so good as the following, the inverted W, though the latter is a little more troublesome to frame, which is sufficient with *some* engineers to condemn it.

In this bent, part (say half ordinarily) of the horizontal thrust H can be combined with the weight resting at each upper apex to find the resultants R_1 and R_2 acting at these apices. If R_1 and R_2 are inside their respective angles, the bent is safe, if the columns are of sufficient strength to sustain the respective components of these resultants.

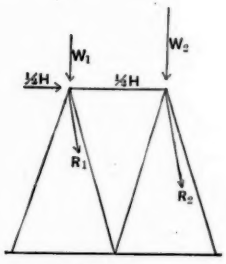


Fig.18

If R_1 passes outside the left post by this decomposition, we must increase the horizontal component of R_2, so that R_1 will give compression on both the posts that sustain it. If both R_1 and R_2 pass outside their respective supporting columns, the bent will be destroyed. It is evident that for the same stability of piers the outside columns can have a less batter than the braces in the preceding figure.

We see how very erroneous it would be to apply the usual method of testing the stability of solid piers to these wooden structures, for such methods suppose the pier to act as one piece in overturning, whereas in the wooden trestles if a destroying force is exerted, the bent will not overturn as a whole, but the posts and braces that would otherwise be in tension pull out of the mortises and the bent collapses by the cap descending sideways to the ground, the posts rotating about their feet as centers.

§ 19.

The previous figures for wooden bents (17 and 18) are used for heights of 10 to 25 feet say. As the only objection to such simple forms is the danger from flexure of the pillars it is very common to spike an X bracing as in Fig. 19, not only as a guard against flexure, but like-

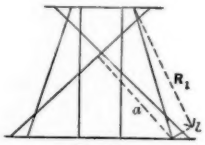


Fig.19

wise to give such solidity to the frame that it would tend to overturn as a whole and not by parts.

Of course this is theoretically a bad type, but it is very efficient practically. The stress on the cross bars may be taken very approximately on the assumption that only one bar acts to resist by tension the overturning effect of R_1 which passes outside the outer brace. If we call l the perpendicular distance from the foot of this brace to the direction of R_1 and a the lever arm of the cross piece about the same point, we have, the stress in the cross piece $= \frac{R_1 l}{a}$.

The size of the piece and spikes that attach it to the posts and sills should be

proportioned to resist this strain. For greater heights of trestle, several bents are superposed, one above the other, forming a 2 *deck*, 3 *deck*, &c., trestle. Figs. 20, 21 and 22 represent forms of

Fig.20

Fig.21

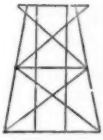


Fig.22

this kind. These types may be extended to any height. The size of the timbers is generally uniform from top to bottom, so that if it is sufficient to carry the whole loading at the top, after previous decompositions, there need be no fear of want of strength in the lower bracing where the strains are divided up amongst a greater number of pieces, so that a strict computation is unnecessary. I suggest Fig. 20, which I have never seen used, as a preferable form to either of the other two.

Trestles are not generally sufficiently braced longitudinally or in the direction of the axis of the road. The most efficient form is X bracing, spiked on and extending from bent to bent. I have been informed that a 4 mile trestle only 15 or 20 feet high, without any longitudinal bracing was—the whole of it—knocked down by a freight train striking it in a certain manner at one end.

I have not mentioned the trestling, whose bents are formed of two piles, 6 to 8 feet apart, projecting out of the ground and capped for the stringers to rest on, because the force of the wind is here principally resisted by the resistance to cross breaking of the piles, though X bracing is generally added both transversely and longitudinally to make a stiffer structure. This form is especially adapted to wide swamps, where a pile driver on a flat car is constantly on hand to repair damages, and likewise to all temporary trestling over soft ground.

From what has preceded, we see that we are much safer in using an approximate solution for wooden than for iron piers with superfluous bars. In fact from the rough manner in which the framing is done for wooden piers or trestles, it would be folly to assume a perfect fit throughout, without which any refinement of calculation is indeed "superfluous."

For iron piers, where it is desirable and practicable to proportion the sizes of the pieces to the stresses they have to bear, the truss without superfluous members is to be recommended, as the unit stresses can be assumed at pleasure; but for trusses with superfluous members, we have seen that it is very rarely the case that they can be made of equal resistance, so that in nearly all cases in practice we should have to assume the sizes of the members and then find the unit stress on each member: then assume other sections that will probably more nearly equalize the unit stresses and so on, until the unit stresses are brought within reasonable limits, even though it may be impossible to give them exactly the values that are preferable. To this difficulty is to be added the influence of misfits, settlement, &c., in altering very materially the computed strains, so that trusses with superfluous members are not to be recommended except in rare cases, for which it is hoped the preceding treatment is sufficiently full to answer the demands of practice.

REMARK.

Since the above was written, an article has appeared in the September number of this Magazine, on "The Resistance of Viaducts to Sudden Gusts of Wind," by Jules Gaudard, translated, &c.

The usual error is made, in ascertaining the stresses in Fig. 4, in not finding the excess of weight thrown on one column of the pier, and the diminution of weight on the other column caused by the wind pressure on truss and train. We have so fully explained the proper method above, that only involves the theory of couples, that it is needless to attempt to make the proof plainer.

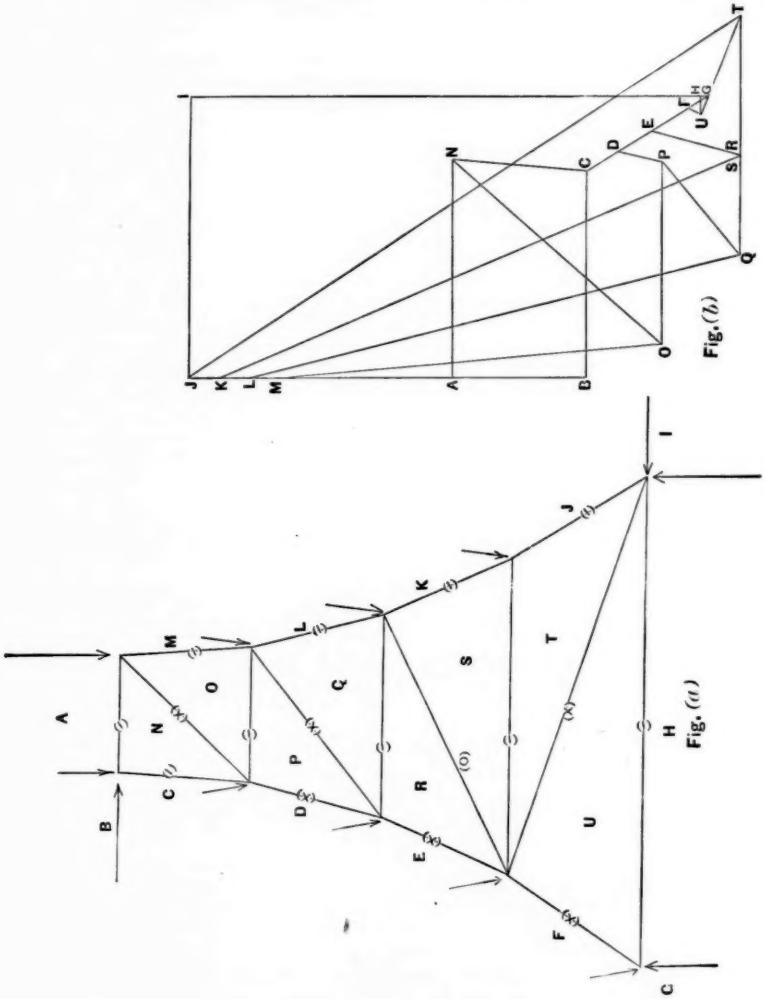
Gaudard gives the horizontal wind pressure on pier from truss at 20 tons, acting at a height of 13.1 feet above top of pier, and the corresponding wind pressure on train as 16.2 tons, acting 27.2 feet above top of pier. As a consequence the excess vertical load borne by the leeward column, is

$$\frac{16.2\times27.2+20\times13.1}{13.5}=52 \text{ tons},$$

and the same load must be subtracted from that due to weight of truss and train borne by the windward column. Now the weight of loaded roadway borne by each column is 51.25 tons, to which add 6 tons for the weight carried at each upper apex, giving 57.25 tons. From this add and subtract 52 tons, giving the resultant vertical load at top of leeward column 109.25 tons, and at the top of windward column 5.25 tons, both acting downwards.

The total wind pressure,

$$16.2+20=36.2 \text{ tons},$$



is transferred now to the top of pier, acting along the top member, by the couple supposed, so that with the other data the stress diagram is quickly drawn.

As a proof of the incorrectness of Gaudard's analysis, he gives as the wind

pressure acting on train 16.2 tons, on truss 20 tons and on pier 20 tons, total 56.2 tons, whereas at the base of pier, he supposes a horizontal reaction of 60.04 tons, which therefore cannot balance the total horizontal wind force as it should.

As the scale is too small to give the stress diagram for this Bouble viaduct very clearly, we append a figure, having some resemblance, with the stress diagram drawn for the forces assumed.

For this pier, the weight supposed concentrated at each apex is combined on the windward side with the corresponding wind force. The other forces are found as before.

The closed polygon of forces is,

J K L M A B C D E F G H I J.

The character of the stresses is as marked on the figure, + for tension, — for compression. This is not so good a form for a bridge pier as one with straight columns of sufficient batter, as a greater number of segments of the windward column are under tension.

THE ELECTRICAL TRANSMISSION OF ENERGY.

By MAURICE LEVY.

Translated from Annales des Ponts et Chaussees for VAN NOSTRAND'S MAGAZINE.

In the transportation of energy, the end to be accomplished is this:—Having at a certain locality A, a permanent source of energy under any form, either mechanical, chemical or calorific, it is desired to utilize it under the same or any other form, at some other place B at any distance from A.

Suppose, at first, that the two points A and B are connected by a simple circuit.

We should place at A an apparatus capable of producing an electrical current by means of the energy existing there. This would be a magneto or dynamo-electric machine if the energy were mechanical; a pile of it were chemical, etc.

At B, on the contrary, we should place an apparatus capable of receiving the current and transforming it into the form of energy we desire to obtain. It might therefore be an electric motor, an electroplating bath, an electric lamp, etc.

Let T_m be the work furnished per second by the apparatus generating the current, and which we will designate the *motor* work, and let T_u be the work afforded per second by the receiving apparatus, and which we will call the *useful* work.

The apparatus A receiving energy becomes the seat of an electro-motive force, such that it reproduces in the circuit exactly the amount of energy received from without.

Now, if in accordance with Joule's law we designate by E the electro-motive force and by I the intensity of the current, supposed constant, the quantity of work per second will be E I. As this is also the work received by A, we have

$$T_m = EI \quad . \quad . \quad . \quad . \quad (1)$$

The apparatus B producing an exterior work T_u becomes the seat of an electro-motive force E′, directed in such way as to lessen the energy of the circuit by the amount of work produced outward. It is necessary then that this force act in a direction contrary to the current. The quantity of energy removed from the circuit will be E′I. Such is also the work produced by the apparatus, and we have

$$T_u = E'I \quad . \quad . \quad . \quad . \quad (2)$$

Furthermore, the action being supposed established, the law of conservation of force teaches us that the motor work is equal to the useful work plus the work expended in heating the circuit. Now if S is the total resistance of the circuit, composed of the resistances of the generator, the receiver and the exterior circuit, the work according to Joule's law is SI^2. Therefore,

$$T_m - T_u = SI^2 \quad . \quad . \quad . \quad (3)$$

These three simple equations are all that is necessary. As has been already shown by the writer in communications presented to the Academy in November, 1881, these equations permit us to study all the important consequences of the

transportation of energy, whatever the form of energy, and whatever the nature of the apparatus or machines employed in the operation. They contain in all six quantities:

$$\begin{array}{ll} T_m & T_u \\ E & E' \\ I & S \end{array}$$

If three of them are known the other three may be found.

Suppose there are given S, the total resistance; T_u the work to be obtained, and E the electro-motive force of the generator. Then the unknown quantities are: T_m, the work of the generator, or $\frac{T_u}{T_m}$ the ratio of the work obtained from B, to the work expended at A (efficiency); the intensity I of the current; and the electro-motive force E′ which is manifested at B.

The values are:

$$\left.\begin{aligned} I &= \frac{E \pm \sqrt{E^2 - 4ST_u}}{2S} \\ E' &= \frac{E \mp \sqrt{E^2 - 4ST_u}}{2} \\ \frac{T_u}{T_m} &= \frac{E'}{E} = \tfrac{1}{2} \mp \tfrac{1}{2}\sqrt{1 - \frac{4ST_u}{E^2}} \end{aligned}\right\} \quad (4)$$

In order that the operation be possible, that is to say, that a current should exist, it is required that

$$S < \frac{E^2}{4T_u}$$

Thus the greatest resistance S, against which a given amount of energy T_u can be transmitted by means of the electro-motive force E, is:

$$S = \frac{E^2}{4T_u}$$

The resistance increases as the square of the electro-motive force of the generator; but this electro-motive force itself cannot be increased indefinitely. There is a limit beyond which the circuit cannot be insulated. Let E_0 be this limit. The corresponding maximum value of S, is:

$$S = \frac{E_0^2}{4T_u}$$

There exists therefore for a resistance against which we can transmit a given quantity T_u of energy, a limit which we cannot pass, however great the mechanical force at our command, and however powerful the electrical motors engaged in the transmission.

Beyond this limit the power of the machine produces no current, nor in consequence any work in the receiving apparatus, but electric sparks along the circuit. In the same manner there exists for the power of traction of a locomotive a limit which depends only upon the weight on the driving wheels and not upon the power of the engine, and beyond which the force exerted by the steam produces only slipping of the wheels, and not motion of the train.

Suppose the value of the electro-motive force to be E, a little less than, or at most equal to E_0. Then the operation would be possible provided that

$$S < \frac{E^2}{4T_u} \quad \text{or} \quad S = \frac{E^2}{4T_u}.$$

If S be taken at this latter value, the preceding equation gives

$$I = \frac{E}{2S},$$

and for the efficiency

$$\frac{T_u}{T_m} = \tfrac{1}{2}.$$

If we take

$$S < \frac{E^2}{4T_u}$$

we get two solutions.

By using the superior sign we get:

$$I > \frac{E^2}{2S}$$

and for the efficiency,

$$\frac{T_u}{T_m} < \tfrac{1}{2}.$$

To obtain the values given by taking the lower signs, it is necessary to reverse the above signs of inequality. The solutions giving real values indicate that we may have a strong current with low efficiency or the reverse.

In the following, the condition of best efficiency will be assumed. Taking therefore the second values, referred to above, we have:

$$\left.\begin{aligned} I &= \frac{E - \sqrt{E^2 - 4ST_u}}{2S} \\ E' &= \frac{E + \sqrt{E^2 - 4ST_u}}{2} \end{aligned}\right\} \quad . \quad . \quad (5)$$

$$\frac{T_u}{T_m} = \frac{E'}{E} = \frac{1 + \sqrt{1 - \frac{4ST_u}{E^2}}}{2} \quad . \quad . \quad (6)$$

From this formula we deduce important conclusions.

It is readily seen from the above that the efficiency is not independent of the quantity of energy T_u to be transmitted. It becomes, other things being equal, less as the energy becomes greater.

Thus when we speak of *efficiency* in the transportation of electrical energy, it is indispensable that the amount of energy to be transmitted should be specified. If twenty-horse power are to be transmitted we shall have, other conditions remaining the same, a lower efficiency than if we get ten-horse power.

It is from a defective understanding of this point, that a correct statement, made by M. Marcel Deprez, in a paper to the Academy of Sciences (March 15th, 1880), has been poorly comprehended, and proved an exciting cause of controversy both during and after the meeting.

After having obtained the expression for efficiency $\frac{E'}{E}$, M. Deprez expressed himself as follows: "A remarkable expression, as it is independent of the resistance of the exterior circuit. It seems extraordinary at first sight, and even contradictory to experience in some cases, unless the conditions of maximum efficiency are fully considered. To make it seem less paradoxical, it will suffice to recall the condition of a current employed to produce energy under another form than that of mechanical work, as for example, that of the decomposition of water in a voltameter. The number of equivalents of water decomposed is always equal to the number of equivalents of zinc dissolved in each of the elements of the battery, whatever the length of the exterior circuit, which, it must be borne in mind, has no influence upon the number of elements necessary to effect this decomposition. Here, then, is a familiar experiment in which the economic performance is not influenced by the external circuit."

It is very true, as M. Deprez says, that whatever the resistance interposed between the battery and the voltameter, a given quantity of zinc consumed corresponds always to the same quantity of water decomposed. But it happens that if the resistance of the circuit becomes ten times as great, the chemical actions are effected ten times more slowly, and the quantity of water decomposed *in a given time*, as a second for example, that is to say, the amount of energy T_u transmitted is only one-tenth as great. But as the quantity of zinc consumed in the same time is also one-tenth as great, the efficiency remains the same.

Faraday also proved that we may maintain the efficiency whatever the distance of transportation, provided that the amount of energy to be transmitted varies inversely with the resistance.

The proposition thus enunciated (and it is thus I think that M. Deprez intended it) is seen to be an immediate consequence of our formula for efficiency (eq. 6). In effect, the electro-motive force of the pile being constant, the efficiency depends only upon the product ST_u of the resistance and the energy to be transmitted. This remains constant, even if the resistance increases, provided the work produced decreases in the same ratio.

This proposition cannot, however, be applied to practical uses. Suppose we have electrical appliances capable of transmitting ten-horse power to the distance of a kilometer, and we wish without losing efficiency to transmit power to a distance of 20 kilometers, or more exactly, against a resistance twenty times as great. The law in question assures us we may do it with the same apparatus, provided that in place of ten horse power we only transmit $\frac{1}{2}\frac{0}{0} = \frac{1}{2}$ horse-power; but as ten-horse power is wanted, the problem is not solved.

Equation 6 shows that the efficiency for a given resistance and given work increases as the electro-motive force E.

The first thing to be determined then with reference to electrical apparatus designed for such work, is the greatest amount of electro-motive force obtainable without injury to the insulation. This

we will call the *available electro-motive force.*

It depends—1st, on the nature and thickness of the insulating material employed to cover the wires of the generating and receiving machines; and 2d, on the nature of the insulation of the conducting wire, which depends upon the character of the supports and varies with the climatic conditions.

This limit, as fixed by these conditions, should be determined for any motor before commencing any important work with it. When once the available electro-motive force is found it should be adopted. To employ a less amount than this thereafter, would be a lack of economy of the same kind as using a steam-boiler designed for ten atmospheres pressure and never employing but two or three.

A first consequence of this important remark is this: since the maximum electro-motive force that can be employed at any locality is approximately determinate, and that from the economical point of view it should be employed for all transmissions great or small; it follows that one or two kinds of machine, designed so that with a suitable velocity this force could be realized, could be employed for all transmissions whatever their importance. We will show further on how this is possible. Such machines once in the market, could be obtained at moderate price.

This same remark leads us to consider a law stated by M. Marcel Deprez in an important paper published in *La Lumiere Electrique*, Dec. 3, 1881.

"The useful mechanical work and the economic efficiency remain constant, whatever the distance of transmission, provided that the positive and negative electro-motive forces vary as the square root of the resistance of the circuit."

I will say in passing that if this law merits this announcement, I believe it proper to claim priority, as it is fairly implied by Eq. 6, which may be found in my communication to the *Academy* in Nov. 7, 1881. It is readily seen that if in this formula all three of the quantities E, E′ and $\sqrt{S}$ vary in the same ratio, whatever that ratio, that neither the efficiency $\frac{E'}{E}$ nor the useful work T_u, will change. This is the law as above stated.

Although this is very interesting from a scientific point of view, it is unfortunately of no use in practice.

Suppose we possess an electrical transmission capable of transporting a certain amount of work, T_u against a resistance S=1 and affording an efficiency of 60 per cent.; we wish to lengthen the circuit and transmit the same work against a resistance of 25 without loss of efficiency. According to the law in question it will suffice to quintuple the electro-motive forces E and E′ adopted in the system. But if the arrangement has been established under proper conditions, we are already employing the highest electro-motive force compatible with the insulation of the circuit. So that to quintuple this force, or even to double it, is out of the question. It is necessary to take it as it is, and to be satisfied with a much lower efficiency in the second case than in the first.

There are many similar laws relating to this class of problems quite exact from a scientific point of view, but unfortunately not available for industrial purposes. Perhaps the following apparent paradox is more singular than any previously referred to:

In the electrical transmission of energy to any amount, not only will the efficiency not diminish as the distance increases, but on the contrary it will increase in direct proportion to the distance, so that if the latter be sufficiently great there would be no sensible loss, provided the electro-motive force of the generating motor be made to increase in proportion to the resistance of the circuit.

Suppose that E increases proportionally to the resistance S, so that

$$I=KS,$$

K being an arbitrary constant. Eq. 6 gives for the efficiency:

$$\frac{T_u}{T_m}=\frac{1+\sqrt{\frac{4T_u}{KS}}}{2}$$

Then as the resistance S increases, the efficiency also increases, although the work transmitted T_u remains constant. And for S=infinity we have an efficiency equal to unity.

But the difficulty of providing ade-

quate insulation amounts to a practical impossibility. So that, in the matter of practical application, this last resembles the one previously discussed.

I propose to show that the laws governing the electrical transmission of force, supposing the currents permanently established, do not differ from those relating to transmission of force through a simple water conduit in which the velocity is moderate and uniform.

Suppose our store of energy at the point A to be that of a fall of water, H feet in height, furnishing P liters of water per second, of which we wish to employ the least possible amount in order to obtain at the point B an amount of work $=T_u$. The motor work is:

$$T_m = PH \quad . \quad . \quad . \quad (1').$$

Let the water start from a tank at A and be delivered through a pipe or conduit to B, where it drives the receiving motor. The loss of work in the conduit and receiving motor is at moderate velocity sensibly proportioned to the square of the velocity, and therefore proportional also to the square of the delivery. The loss may then be represented by SP^2, in which S is a constant depending upon the size and nature of the conduit and the receiving motor.

If T_u is the work afforded at B, then the theorem of living forces gives

$$T_m - T_u = SP^2 \quad . \quad . \quad . \quad (3').$$

Finally, if $H - H'$ is the loss of head between A and B; then we have

$$T_u = PH' \quad . \quad . \quad . \quad (2').$$

The three equations (1'), (2'), (3'), are identical with (1), (2), (3), with the difference that P, H, and H' have replaced IE and E'. We can deduce, therefore, the same consequences and the same laws.

We will now seek the solution of the problem of electrical transmission of any given amount of energy to any distance, to obtain any desired efficiency without destroying the insulation.

Let a be the efficiency to be obtained. Eq. 6 gives:

$$\frac{1+\sqrt{1-\frac{4ST_u}{E^2}}}{} = a, \quad . \quad . \quad . \quad (a).$$

from which we get

$$S = \frac{E^2}{T_u} a(1-a).$$

As T_u is given, we see that with a given efficiency we can transmit against a resistance that increases as the electro-motive force increases. Taking for E the maximum value E_o as before used, then

$$S = \frac{E_o^2}{T_u} a(1-a).$$

Then for any value of a, the maximum resistance against which work can be transmitted is determinable, and if we wish an efficiency very near unity, this resistance will become extremely small. The problem then is this:

For a given distance of transmission, can we, if this distance is very great, make the resistance as small as we wish?

Now the total resistance is made up of the resistances of the generator, the receiver and the external circuit, which we will express by

$$S = \rho + \rho' + R.$$

This last term may be rendered very small, even for great distances, by employing a large conducting wire for the external circuit. It is only a question of expense. There is no impossibility in the matter.

In regard to the resistance ρ of the generator. If we reduce this resistance, the machine will no longer furnish the electro-motive force E_o which we require; and similarly if the resistance ρ' is made too small, the receiving motor will no longer furnish the electro-motive force $E' = aE_o$ which we require to make the efficiency $\frac{E'}{E} = a$, unless we construct machines of colossal dimensions for slight transmissions.

Of the three quantities therefore which compose S, one only can be made very small, and consequently the problem is not soluble, at least with such a circuit as we have been considering.

But the problem is nevertheless capable of solution by simple means, which I will proceed to indicate. Take near the connections of the machine A two points; connect by n equal wires and place on each a machine identical with A, each therefore capable of producing an electro-motive force E_o.

In the same manner, in place of the receiving motor, take n' receivers located upon lines all uniting in two points upon the principal circuit.

The intensity of the principal circuit being I, that of each of the derived lines will be $\frac{I}{n}$; the motor work expended for each generator will be $E\frac{I}{n}$, and the total motor work remaining always

$$T_m = EI \quad . \quad . \quad . \quad . \quad (1'').$$

In the same manner the useful work obtained will be

$$T_u = E'I \quad . \quad . \quad . \quad (2'')$$

E′ being the electro-motive force of each receiver.

Furthermore, Ohm's law applied to a closed circuit between one of the generators and one of the receivers, will give:

$$E - E' = \rho \times \frac{I}{n} + \rho' \frac{I}{n'} + RI$$

or

$$E - E' = S'I$$

by making

$$S' = \frac{\rho}{n} + \frac{\rho'}{n'} + R,$$

and multiplying by I,

$$T_m - T_u = S'I^2 \quad . \quad . \quad . \quad (3'').$$

The three equations (1″), (2″), and (3″), differ from (1), (2), and (3), only in the fact that S is replaced by S′. All the consequences thus deduced with one value of S may be realized with the other. Terms which form S′ may be made as small as we wish; R by making the exterior circuit sufficiently large, and the other two terms by making n and n' sufficiently great.

The problem proposed, therefore, of transmitting any desired amount of energy to any distance and obtaining a given efficiency, is capable of both theoretical and practical solution.

The solution of the problem may be effected in a more economical way by exciting the separate machines upon the derived circuits, thus reducing the number of machines, which, of course, is easily conceived.

The arrangements thus indicated by our theory may be practically realized and are the best we could adopt. But the preceding theory assigns no limit to the operation; that is to say, according to it, it would be really possible to transmit to any distance an amount of energy so great as to yield any desired efficiency; provided we have 1st, a sufficient number of machines, and 2d, a sufficiently large conductor for the exterior circuit.

But it is not to be expected that in practice such a result can be completely realized, by reason of the influence of the extra currents due to the periodicity of the principal currents — an influence which we have neglected to consider, but which becomes rapidly greater as the length of the circuit increases. We have neglected, also, the currents produced in the soft iron cores of the machines. We reserve the discussion of these two important points.

The conclusions then are: 1st, the problem of the transmission of a given amount of energy to any given distance, with a given degree of efficiency, finds no real solution in the laws above stated. The laws scientifically exact are illusory in practice, because their application requires either an increase without limit of the electro-motive force, which would render insulation impossible; or else a decrease indefinitely of the energy transmitted, which would render the operation useless.

2d. But the problem may be resolved theoretically without limit; practically, under the conditions just stated above, by the employment of machines of ordinary size and uniform type for all transmissions whether of greater or lesser amount; thus rendering the cost low and the replacement easy. It will suffice then to join a greater or less number of these machines (for quantity not tension) according to the work to be performed.

3d. We can reduce the number of the machines in combination, described above, by exciting directly some of the machines in the branch circuits.

4th. It results from the above conclusions there is no object gained, so far as the transmission of force is concerned, in the construction of colossal machines like that, for example, which Mr. Edison exhibited at the Exposition of 1881. Not only will machines of ordinary dimensions solve the problem by the dis-

position above proposed, but they have furthermore this advantage when placed in separate branch circuits; if one becomes temporarily disabled, the others continue their work and even supply the deficiency by an elevation of the tension.

5th. In order to establish types of machine practically useful for all kinds of transmission, it will be necessary to first try some practical experiments, easily devised, in order to determine the maximum tension to which, in all seasons, an aerial or a subterranean line can be subjected.

The machines should be such as to afford this tension without being driven at too great velocities. The calculations by which such machines would be determined are analagous to those in our communications to the Academy Nov. 14th and 21st, 1881, except for the points mentioned below.

By the employment of such machines under the conditions specified, we may regard the problem of transmission to any distance, of energy to any amount, as solved, subject (*a*) to the difficulties of the second order which may present themselves in practice and which are always conquered; and (*b*) what is more important, the modifications to which the results of the formulas must be subjected to allow for the periodicity of the currents, and the self-induction of the currents among themselves; also the currents produced in the soft iron armatures and which absorb a certain quantity of work. These two phenomena, whose effects may be quite sensible, should cause us to regard the solutions here given as only first approximations. And in applying in practice any formulas based on the absolute permanency of currents, and the abstraction of currents which have their origin in iron magnets, we ought, as in the case of resistance of materials, to refrain from indulging in the hope of realizing even for a long time the extreme results which these formulas indicate.

The discussion, however, is none the less important and useful. It has furnished us upon the essential points of the problem of electrical transmission of energy with some precise ideas of a general character; that is to say, independent of the nature of the energy transmitted and of the kind of machines employed.

It has permitted us to destroy some erroneous ideas regarding efficiency, ideas which had become to some extent convictions in the public mind. It has led us furthermore to the most favorable practical arrangements, the closest study of the phenomena relating to the causes of perturbation above mentioned, modifying in no essential point this disposition of the parts, but only proving that the useful effects are not as unlimited as an unreasoning confidence in the formulas might lead one to believe; formulas of first approximation only which have been the object of this essay, and the completion of which we reserve for the future.

REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—The last number of the Transactions contains:

Paper No. 240.—On the Determination of the Flood Discharge of Rivers and of the Back Water caused by Contractions. By Wm. R. Hutton. With discussions on the paper by Theodore G. Ellis, Robert E. McMath, and Wm. R. Hutton.

Paper No. 241.—Accuracy of Measurement increased by Repetition. By Stephen S. Haight.

ENGINEERS' CLUB OF PHILADELPHIA.—The latest issue of the Proceedings contains:

No. 3.—Applications of Logarithms to Gearing. By Wilfred Lewis.

No. 4.—Working Strength of Bridge Posts. By Geo. P. Bland.

No. 5.—Thickness of Metal for Cast Iron Pipes. By P. H. Baermann.

No. 6.—Resistance to Traction on Roads. By Rudolph Herring.

No. 7.—Philadelphia and Long Branch Railway. By C. S. d'Invilliers.

No. 8.—Brickwork under Water Pressure. By D. McN. Stauffer.

ENGINEERING NOTES.

Two distinct rock drills are used in the Arlberg Tunnel. That on the east side is the Ferroux drill, which has rendered such good service in the St. Gothard; and that on the west the Brandt rotary perforator, which works by water under pressure. It has already given good results at Pfoffensprung, and the inventor guarantees a minimum advance of 2 meters a day, which has been considerably exceeded. The motive power is obtained by water wheels erected in the valley which separates the two slopes of the Arlberg. The following figures give the progress from the commencement, 17th November, 1880, to the end of February last: Advance of heading, 320

meters=350 yards; mean daily advance, 3.07 meters=10 feet; number of blasting operations, 295; advance for each operation, 1.08 meters=3 feet 6 inches; number of shots in each operation, 19; weight of dynamite used for each meter of advance, 22 kilogrammes=say 44 lbs. per yard.

A MASSACHUSETTS paper states that the Railroad Commissioners have received at their offices, in Pemberton Square, an instrument, by Dr. Thomson, of Philadelphia, which is in use for the detection of color-blindness upon the Pennsylvania Railroad. The invention suggested itself to Dr. Thomson from the fact that the number of employes upon the Pennsylvania system of railroads comprised upwards of 35,000 persons, scattered over more than 2500 miles; and as the number of trained ophthalmic surgeons was limited, it was desirable to find a system which would enable the facts to be collected by any intelligent employe in the company's service in such a form as to enable decisions to be justly made by scientific experts, although personally absent from the examination. The instruments used consist of two flat sticks, abont 2 feet in length and 1 inch in width, fastened by a hinge at one end and connected by a button at the other. Between them, and concealed from view, are forty white buttons, having the figures from 1 to 40 upon them, attached to the stick by small wire hooks, which permit of easy removal or change of position. To the shanks of these buttons are attached forty skeins of colored wool. The test skeins are separate, and three in number—light green, rose or purple, and red. These skeins are shown to the persons examined in turn, and they are directed to select from the stick the colors which will match them. When the examination is made the instrument is closed to conceal the number, and test greens being shown, the person examined is directed to select ten tints from the stick; and when this is done the figures are recorded by the clerk, and the selections thus made can be identified at any future time. After a protracted experience upon several thousand employes of the Pennsylvania Railroad, that company has adopted the invention, and it will be used for examinations hereafter.

RAILWAY NOTES.

A TRAM-CAR axle has been recently patented by a Dane, the object of which is to allow the wheels to pass round sharp curves without grinding. For this purpose the axle is divided in the center, the end of one-half having a hollow, and that of the other a corresponding projection, somewhat similar to a ball and socket joint, the necessary stiffness being given to the axle by a tube which surrounds the axle and extends between the naves of the wheel, against which it bears by gun-metal collars At the center, between the tube and the axle is a gun-metal bearing, in which the axle can revolve. The wheels act in such a manner that in running along a straight line the wheels and axle turn together, as in an ordinary pair of wheels, but on passing round a curve the axle slips round in its joint, so that the wheel on the inner radius of the curve is retarded and the outer wheel accelerated in proportion to the sharpness of the curve, greater smoothness being obtained in the vehicle, and less wear and tear of the tire and rail.

RAILROADS OF THE UNITED STATES.—Taking the whole system of which "Poor's Manual" has information, the following comparisons are shown of 1881 with 1880:

	1881.	1880.	Increase.	P. c.
Miles of road in operation	104,813	95,455	9,358	9.8
Miles of sidings and second track	26,211	21,978	4,233	19.2
Miles of steel tracks	49,063	33,680	15,383	42.7
No. of locomotives	20,116	17,949	2,167	12.1
No. of passenger cars	14,548	12,789	1,759	13.8
No. of baggage, mail and express cars	4,976	4,786	190	4.0
No. freight cars	648,295	539,355	108,940	20.2

The capital and earnings of the roads reporting (the mileage being that of the roads reporting for a fiscal year to the Manual, and so not including the road not completed till near the close of the year) are given below:

	1881.	1880.	Increase.	P. c.
Miles reporting	94,486	84,225	10,261	12.2
Stock and debt	$ 6,010,389,579	$ 4,879,401,997	$ 1,112,987,582	22.7
Freight earnings	551,968,477	467,748,928	84,219,549	18.0
Passenger earnings	173,356,642	147,653,003	25,703,639	17.4
Total earnings	725,325,119	615,401,931	109,923,188	17.9
Expenses	448,671,000	360,208,495	88,462,505	24.6
Net earnings	276,654,119	255,193,436	21,460,683	8.4
Dividends	93,344,200	77,115,411	16,228,789	21.8

The capital, earnings, etc., per mile of road of the railroads of the United States as reported in "Poor's Manual" for eleven successive years have been:

Year	Stock and debt. $	Gross earnings. $	Expenses. $	P. c. of ex. to earn.	Net earn. $	P. c. of net earn. on capital.
1871	59,726	9,040	5,863	64.8	3,177	5.32
1872	55,116	8,116	5,224	64.4	2,892	5.25
1873	57,136	7,947	5,172	65.1	2,775	4.86
1874	60,944	7,513	4,776	63.6	2,737	4.49
1875	61,533	7,010	4,425	63 1	2,585	4.20
1876	60,791	6,704	4,228	62.5	2,536	4.16
1877	61,650	6,382	4,075	63.8	2,307	3.74
1878	59,040	6,232	3,847	61.7	2,385	4.04
1879	58,070	6,244	3,670	58.8	2,610	4.49
1880	60,650	7,307	4,277	58.5	3,030	5.00
1881	63,611	7,677	4,749	61.9	2,928	4.60

Gross earnings, we see, fell off every year from 1871 till 1878, and have risen since 17 per cent. from 1879 to 1880, and 5 per cent. from 1880 to 1881. Expenses decreased yearly from 1871 to 1879—one year longer than earnings—but have advanced in the last two years nearly as much as they had fallen in the previous five years. Net earnings have varied much less than gross earnings; but they fell from 1871 to 1877, and then rose for two years, but fell off last year again, remaining larger, however, than in any previous years, except 1871 and 1880. Of the proportion of net earnings to capital, we have already spoken.—*Railroad Gazette.*

THE extent to which the manufacture of locomotives is now carried on in the United States may be gathered from the figures given below, which we take from Mr. Drummond's report. There are now 15 locomotive works in the United States, with a capacity of from 8 to 50 engines per month. In 1881 they turned out in round numbers 2,700 locomotives. Add to this 300 built by railway companies, and we have at least 3,000 new engines constructed during the year, besides those rebuilt. At the commencement of last year there were, speaking roughly, 18,000 locomotives running on the 94,000 miles of railway in the Union, or an average of about one engine to every five miles. If, as is probable, the new railway construction this year reaches 10,000 miles, this average would call for 2,000 new engines. The life of a locomotive is estimated by manufacturers to average from fifteen to twenty years. The latter figure is probably more nearly correct, as the improved condition of American railways has prolonged the existence of engines considerably. At this rate about 1,000 new engines per year would be required to keep good the reduction by decay. Adding this to the 2,000 presumably required this year for the increased mileage, we find that about 3,000 new engines will be demanded. The great Boston statistician, Mr. Atkinson, believes that in the next sixteen years there will be added 100,000 miles of rail. They deal in big figures over the water.—*Engineer*.

ORDNANCE AND NAVAL.

THE NEW GERMAN MAGAZINE GUN.—This weapon, which is considered by the German Government to have proved itself the most suitable military repeating rifle, is the invention of Messrs. Mauser, the originators of the present German regulation rifle. The magazine consists of a tube contained in the stock, and has a spiral spring which keeps the cartridges up to the breech action. When the bolt is withdrawn, a cartridge—which has been forced out of the magazine by the spiral spring—is raised up to the level of the cartridge chamber, into which it is driven by the bolt as it returns. The whole action of loading is comprised in the backward and forward motion of the bolt. In order to avoid waste of ammunition, a lever is attached to one side of the action, by which the magazine can be instantly closed, the gun being then loaded and fired as an ordinary breechloader. The reloading of the magazine is stated only to occupy a few seconds. This system can be applied to the Mauser rifles of 1871 model now in use, at very small cost. Two thousand of these weapons are in the course of construction, and will be served out quickly as possible to one of the grenadier regiments now quartered in Spandau.

IRON AND STEEL NOTES.

TWO inventors in Bohemia have patented a process for enameling cast iron water pipes, which can be applied to other hollow castings that are made with cores. It consists, the *Building News* says, in simply covering the sand core with the enamel and then pouring in the iron as usual. The heat of the melted iron fuses the enamel, which attaches itself firmly to the iron, and detaches itself so completely from the sand that the enamel is said to be all that can be desired for water pipes and other industrial purposes. In casting sinks, basins, urinals, &c., the enamel can be applied to the sand on that side of the mould which is to form the inside of the basin. The composition of the new enamel is kept a secret, but is said to differ from the old form in the simplicity of its preparation and the extraordinary cheapness of the materials used. In color this new enamel is gray. It will be useful for gas pipes, and soil pipes as well as water pipes, because it will make the pipes absolutely tight by a glassy lining.

PAINTING IRON SURFACES.—Continually growing in importance as iron becomes more and more an every-day building material is the best method of preserving it by paint, The various chemical methods of rust-prevention being as yet too imperfect and too expensive for ordinary use. The following extracts from a paper read by Mr. William Meeking, before the Civil and Mechanical Engineers' Society, London, furnishes some technical points of interest in relation to this subject. It says :

Of the varieties of lacquers and paints used it is needless to speak at length as the all-important point is the actual state of the iron surface when the first coat is laid on. If that is not in proper condition no subsequent application, however good in itself, has any chance of being permanently preservative, and I think that that proper state is found when there has been formed upon the whole surface of the work a thin layer of the first or black oxide, which has been, while hot, thoroughly permeated by and incorporated with a resinous and tarry covering. Once formed, everything goes well. Additional coats of paint may be applied from time to time to renew the thickness of the original covering, but the iron underneath remains unattacked. If, on the contrary, a film of hydrate oxide (ordinary rust from exposure) be once allowed to form, the successive coats of paint are thrown off sooner or later, and, in the meantime, the rust has spread under the paint. A striking instance of this may be generally seen after outdoor riveted work has been in place for some time. As a rule all the riveting is done before the final painting is commenced, and each rivet-head has in the mean time been exposed to a damp atmosphere ; the paint invariably commences to peel off the rivet heads long before it leaves the adjacent plates, and when this has once taken place nothing but a thorough scraping off of the surface will give the paint any chance of adhering. So slight are the differences of manipulation which determine whether a given piece of work shall or shall not rust away, that I think they may all be found in the different methods of manufacture pursued now and formerly. Taking the case of a piece of ornamental iron work, which in so many instances has come down to us in unimpaired beauty and condition, it would be

now probably forged in detail in one part of a factory, drilled, filed and fitted in another, and when completely finished be painted "in three coats of best oil paint." Formerly the smith who forged the work punched the necessary holes at the same time, fitted his various pieces together as he went on, completing each piece as he proceeded, doing all the work with his hammer, and, to quote an old book of direction to good smiths, "brushing his work over with linseed oil, and suspending it for some time over a strongly smoking wood fire." This will give at once a sort of elastic enamel coat, perfectly adherent, calculated to preserve the iron to the utmost.

To come to practical uses: it appears to me, first, that in all cases where iron is used externally there should be the most careful provision made for draining off water, and preventing any lodgment in inaccessible places; second, that the iron used should be in the largest and most compact masses possible, with a due regard to the necessities of construction, avoiding, by all means, such designs as are calculated to provide the largest possible surface for a given weight of metal; third, to take care that, before the metal leaves the iron works, and while heated, it receives a coat of some protective substance, such as tar or linseed oil, which shall be allowed to incorporate itself with its external surface and form a durable substratum for future coverings.

BOOK NOTICES.

PUBLICATIONS RECEIVED.

REPORT OF THE BOARD OF COMMISSIONERS OF THE NINTH CINCINNATI INDUSTRIAL EXPOSITION.

REPORT OF NEW YORK STATE SURVEY FOR THE YEAR 1880. James T. Gardner, Director.

CONSTITUTION, BY-LAWS AND LIST OF MEMBERS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS.

ASTRONOMY CORRECTED. By H. B. Philbrook. New York: J. Polhemus.

REFINING AND SEPARATING THE METALS CONSTITUTING BASE BULLION BY THE ELECTROLYTIC PROCESS. By N. S. Keith.

MONTHLY WEATHER REPORT FOR JULY. Washington: Government Printing Office.

MODERN APPLICATIONS OF ELECTRICITY. By E. Hospitalier. Translated by Julius Maier, Ph.D. New York: D. Appleton & Co. Price $4.50.

Five years ago, as the translator of this book says, "a work like the present would have had no *raison d'etre;* at this moment it requires no introduction and no recommendation."

What the reader chiefly desires to know is: how completely does it fulfill the implied promise of the title? To which it may be answered, that the original treatise was completed last year by an unquestioned authority in electrical science, and who had enjoyed exceptional advantages for gathering the necessary information.

In addition to which it should be said that the translator carefully compiled and added an account of the discoveries in practical electrical science made during the year which has intervened, only completing his work in April last.

456 pages of text are illustrated by 170 good wood-cuts.

The non technical reader can understand it all.

OLMSTEAD'S COLLEGE PHILOSOPHY. Third Revision, by Rodney G. Kimball, A.M. New York: Collins & Brother.

For many years Olmstead's Philosophy has held a deservedly high place among American text-books. The successive editions have been revised by able writers, who have incorporated in their work the later discoveries, so that notwithstanding an increasing number of competitors, the book is still considered by prominent instructors as the best compend of the fundamental principles of physical science, and moreover, the book best fitted for the purposes of instruction.

Many teachers and students throughout the country will gladly learn that the book will in no wise lose prestige by this last revision. With a full appreciation of the merits which had previously insured success, and with the talents of a successful teacher of applied science, Prof. Kimball has brought this favorite text-book abreast with modern science, and made it again a sufficient course of Physics for high schools and colleges.

The new edition is necessarily somewhat larger than the previous one. New sections and new illustrations were indispensable. The concise, logical and accurate method of presenting the *principle* characterizes the new portion as it did the old.

CONTINUOUS RAILWAY BRAKES. By Michael Reynolds, London: Crosby, Lockwood & Co.

The author's preface says, "I have endeavored to explain from my experience what a continuous brake should be capable of doing, and when it is found most useful.

"I have given cases to show that a continuous brake in the hands of the driver would, in all probability, have saved the lives of passengers who were killed. With such evidence before us, every accident which takes place in the future with fatal results will, no doubt, be subjected to rigorous investigation. * * *

"I have endeavored to illustrate continuous brakes for the ordinary reader, at the same time adhering closely to technical details of construction for the professional reader."

The brakes illustrated and described at length are the screw brake, chain brake, Smith's vacuum brake, Hardy's vacuum brake, Steel & McInnes' compressed-air brake, Eames' continuous vacuum brake, Aspinwall's automatic vacuum brake, Barker's hydraulic continuous brake, Sanders' vacuum brake, and the Westinghouse automatic brake.

L'Electricitie et ses Applications.—Henri de Parville, Paris: G. Masson.

This is a popular and well illustrated account of the Paris Electrical Exhibition.

Beginning with a discussion of the nature of electricity, the author passes quite directly to the methods by which it is produced. Then comes the transmission of energy, electric lighting, telephones and microphones; the latter especially receiving an undue share of attention.

The illustrations which are good and abundant will look familiar to readers who have read the current literature on applications of electricity.

The Metal Turner's Hand-book. By Paul N. Hasbuck, London: Crosby, Lockwood & Co. Price, 40 cents.

This useful little treatise is designed for amateur workers at the foot lathe.

Lathes are treated first, then gearing attachments, slide rests, chucks, cutters, tool grinding and finally lathe motors.

The author wastes no words in his descriptions. The illustrations are very numerous, there being one hundred figures for one hundred and fifty pages of text.

Any one owning a foot lathe will find this little book worth the price demanded for it.

The Laboratory Guide: A Manual of Practical Chemistry, for Colleges and Schools, Specially Arranged for Agricultural Students. By Arthur Herbert Church, M.A., of Lincoln College, Oxford. Fifth edition, revised and enlarged. London: John Van Voorst.

On comparing the present edition of Prof. Church's Laboratory Guide with its earlier phases, we cannot fail to be struck with the great changes which have been made. Whilst the general plan of the work has been retained, and whilst none of the features which won for it the general approval of teachers and students have been sacrificed, additions and improvements have been numerous.

The chapter on the analysis of drinking-water has been greatly enlarged and modified. It is very satisfactory that Prof. Church does not consider that the character of a water can be deduced from two or three data alone, but considers it advisable to ascertain the presence or absence of phosphoric acid, to observe the action of the water on lead, to apply Heisch's sugar-test, and to submit the deposit to microscopic examination. He does not refer to the presence or absence of free oxygen, which is in some cases an important feature.

The instructions for the determination of the albumenoids in articles of diet, form an exceedingly useful addition in the present volume as compared with the earlier editions. Until a comparatively short time ago it was believed that the nutritive value of any root, leaf, &c., could be discovered by a simple determination of its total nitrogen. It is now known that nitrogen can and does exist in forms in which it is not capable of assimilation by the animal system. Hence a determination of the albumenoids becomes necessary. Two methods for this purpose with carbolic acid and with copper hydrate are accordingly given.

Prof. Church's work as it stands is undoubtedly the best laboratory guide which can be put into the hands of the agricultural student,—a class whose requirements extend far beyond that mere valuation of manures and soils of which they are popularly supposed to consist.—*Chemical Review.*

MISCELLANEOUS.

A New Electro-Dynamometer.—At the meeting of the Physical Society of London, which was recently held at the Clarendon Laboratory, Oxford, an electro-dynamometer, which has some novel points of construction, was exhibited by Dr. W. H. Stone, F.R.S. It was designed for measuring the currents used in the medical applications of electricity, and originated in a suggestion of Mr. W. H. Preece, made at the Society of Telegraph Engineers, when Dr. Stone read a paper on "Medical Electricity," which we referred to in a recent note. The chief novelty in the new instrument is the use of aluminium wire instead of copper for the suspended coil. Aluminium is chosen because of its lightness as compared to copper, and its equal conductivity to copper, weight for weight. In an electro-dynamometer the movable coil ought to be as light as possible, other things being the same, as it plays the part of a needle and is deflected by the current just as the aluminium needle of a quadrant electrometer is deflected by the difference of potentials between the quadrants. The aluminium coil of Dr. Stone was made from silk-covered wire prepared by Messrs Johnson and Matthey, and is wound into without a frame, the convolutions being bound together by small ties of silk and a lacquer of amber varnish such as is used by photographers. Dr. Stone recommends this varnish for delicate electrical uses instead of the ordinary shell-lac varnish. The coil is suspended from two fibers of silver gilt wire, such as is used in gold-lace making. This wire is gilt before it is drawn, and has a high conductivity. Thus a meter of wire $\frac{1}{100}$ in. in diameter measures 9.8 ohms, whereas a platinum wire of the same length and thickness measures 62.2 ohms. As the current is conveyed to the suspended coil by this wire, it is important to have it of low resistance. Moreover, the gilt surface makes a good clean contact with the aluminium wire of the coil, and thus overcomes one of the leading obstacles in the way of using aluminium wire for electrical purposes. Dr Siemens and others have tried to use aluminium before, but the difficulty of getting a good soldered joint was found to be a drawback. The gold and aluminium clamped together or soldered after the aluminium is electro-plated with a solder-holding metal, is likely, however, to answer the purpose. Aluminium has also a high specific heat, and is very difficult to fuse, therefore it is adaptable for resistance coils. The bifilar suspension is necessary in Dr. Stone's instrument to give the coil a directive force and bring it back to zero. The silver-gilt wires are hung from two brass springs placed horizontally and opposite each

other. These springs can be drawn apart if need be by means of adjusting screws in order to vary the sensitiveness of the needle. The instrument is small in size, and of a portable construction.—*Engineering.*

PROFESSOR H. M. PAUL has communicated to the Seismological Society of Japan some notes on the effect of railway trains in transmitting vibrations through the ground. A box, holding about 20 lbs. of mercury thickened by amalgamation with tin, was placed upon a heavy plank screwed to the top of a post sunk 4½ ft. into the ground. Images reflected in the surface of the mercury were observed by a telescope, as in meridian observations. An express train passing at a distance of one-third of a mile, set the surface of the mercury in confused vibration for two or three minutes. The experimenter, *Nature* says, also found that a one-horse vehicle passing along a graveled road 400 ft. or 500 ft. distant caused a temporary agitation of the mercury whenever the wheels struck a small stone.

INSTEAD of the methods of testing and comparing hardness at present in use, Dr. Herz, of Berlin, has sought a more absolute method, and he has confined himself, on account of the complexity of the question, to the consideration of isotropic elastic substances. In these the hardness may be determined by the pressure which must be exerted on a round mass to exceed the limit of elastic resistance. In the case of plate-glass, *e.g.*, it was found by experiment that, at a pressure of 136 kilogrammes per square millimeter, the limit was passed, and a circular crack was produced; 136, accordingly, expresses the degree of hardness of the glass. Every isotropic body which has its limit of elasticity exceeded under greater or less pressure is, respectively, harder or less hard. The advantage of this method lies in the fact that no second substance is needed, but only two specimens of the substance examined.

A SMALL international industrial exhibition is being held at Lille, under the auspices of the municipal authorities. The exhibitors, are chiefly French and Belgian, but there are two English, viz., Doulton and Minton, ceramic ware being one of the classes. A prominent feature is the artistic ironwork, produced entirely by the hammer, and black, relieved by polished steel, nickel, and copper, which produce an excellent effect; fine scroll-work, flowers, and fruit are marvelously executed. One of the Dandenné perpetual clocks, like that at the Northern Terminus, Brussels, is erected outside the building. It is kept going by the weights being kept constantly wound up by a fan actuated by the ascensional current of an air tight shaft; and when the weight nears the top of its course it puts on a brake which stops the fan, provision being made for twenty-four hours' working in the event of a temporary cessation of the current. Some original improvements in mechanical drawing appliances are shown by M. Jardez, of Lille. He stretches the paper by a panel secured by iron bars. The left-hand edge of the board is provided with a scale and also with a grooved rod, fixed by pins, on which the square works for dispensing with a true edge. The stock of the T-square has an aperture for adjustment, and the blade is also graduated. There is besides a small rack for hatching regularly. Other novelties are folding iron trestles and some metallized cloth for roofing purposes.—*Engineer.*

AT a meeting of the Cleveland Institution of Engineers, held at Middlesbrough on Monday evening, the 12th inst., Mr. J. E. Stead, F.C.S., read a paper "On the Rapid method of Estimating Phosphorus." He described the old method of testing for phosphorus, which occupied two days for each estimation. He then explained the new plan he had devised, whereby the same results can be obtained in two hours. In testing for phosphorus in basic steel, there is a special advantage in dealing with such material, because it contains no silicon, and under such circumstances the phosphorus can be determined in a single hour. The principal saving of time arises from the absence of any necessity for artificial drying. Mr. Stead then read another paper upon a new apparatus designed by himself for analyzing blast furnace gases. The apparatus is in two portions—one portion being used for collecting samples of gas from the mains, and the other portion for dealing with it in the laboratory. Mr. Stead stated that during the production of one ton of pig iron combustible gases weighing nearly 7 tons pass off from a Cleveland blast furnace, and that the calorific power of these gases is equal to that furnished by the combustion of 11½ cwt. of coal. In the production of one ton of pig iron, 5½ tons of air are forced into the furnace, and the combustible gases drawn off from the top of the furnace require 4¾ tons more air to complete their combustion. The total final products of combustion weigh 11¾ tons, and these pass into the atmosphere as waste gases. Mr. Stead advocated strongly the systematic examination of blast furnace gas, stating that he had occasionally detected that one-third of the combustible gas produced was passing into the atmosphere unconsumed. This was equivalent to throwing away about 70 tons of coal per week for each furnace producing 400 tons per week of pig iron.—*Engineer.*

A NEW explosive has been invented by M. Petri, a Viennese engineer. The name given to it is dynamogen, and, according to the *Neue Militarische Blatter*, it is likely to compete seriously with gunpowder. The inventor states that it contains neither sulphuric acid, nitric acid, nor nitro-glycerine. The charge of dynamogen is in the form of a solid cylinder, which can be increased in quantity without being increased in size, by compression. The rebound of the guns with which the new explosive has been tried is said to have been very slight. It is also said that the manufacture of dynamogen is simple and without danger, that it preserves its qualities in the coldest or hottest weather, and that it can be made at 40 per cent. less cost than gun powder.